How warm are non-thermal relics?
Out-of-equilibrium dark matter production

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Dark Matter 2021

Based on [**arXiv:2011.13458**] – **JCAP 2021**
in collaboration with **G. Ballesteros** & **M. A. G. Garcia**

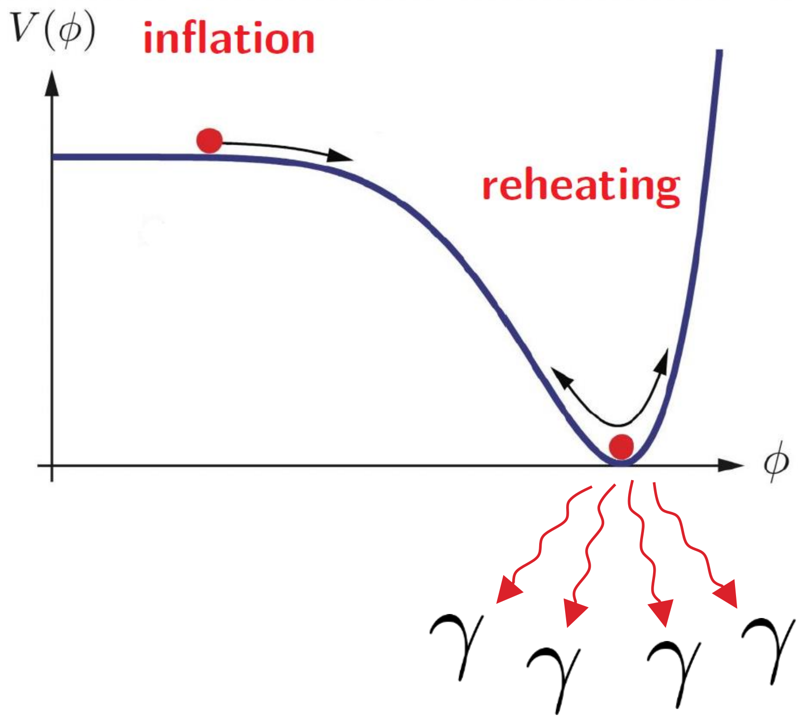
How dark matter is produced?

Can we probe the production mechanism?

➡ **Focus on out-of-equilibrium particle dark matter**

Production of out-of-equilibrium dark matter

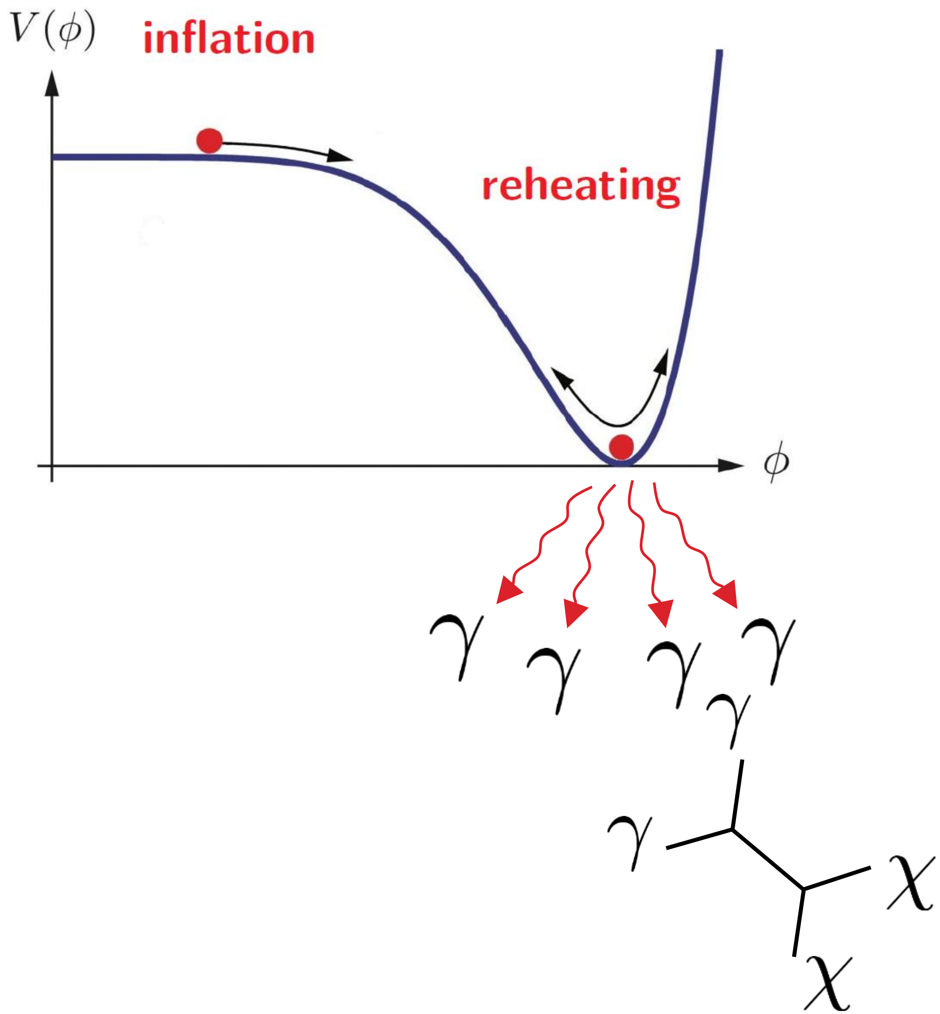
Out-of-equilibrium DM production



ϕ : **inflaton**

γ : **generic SM particle**

Out-of-equilibrium DM production

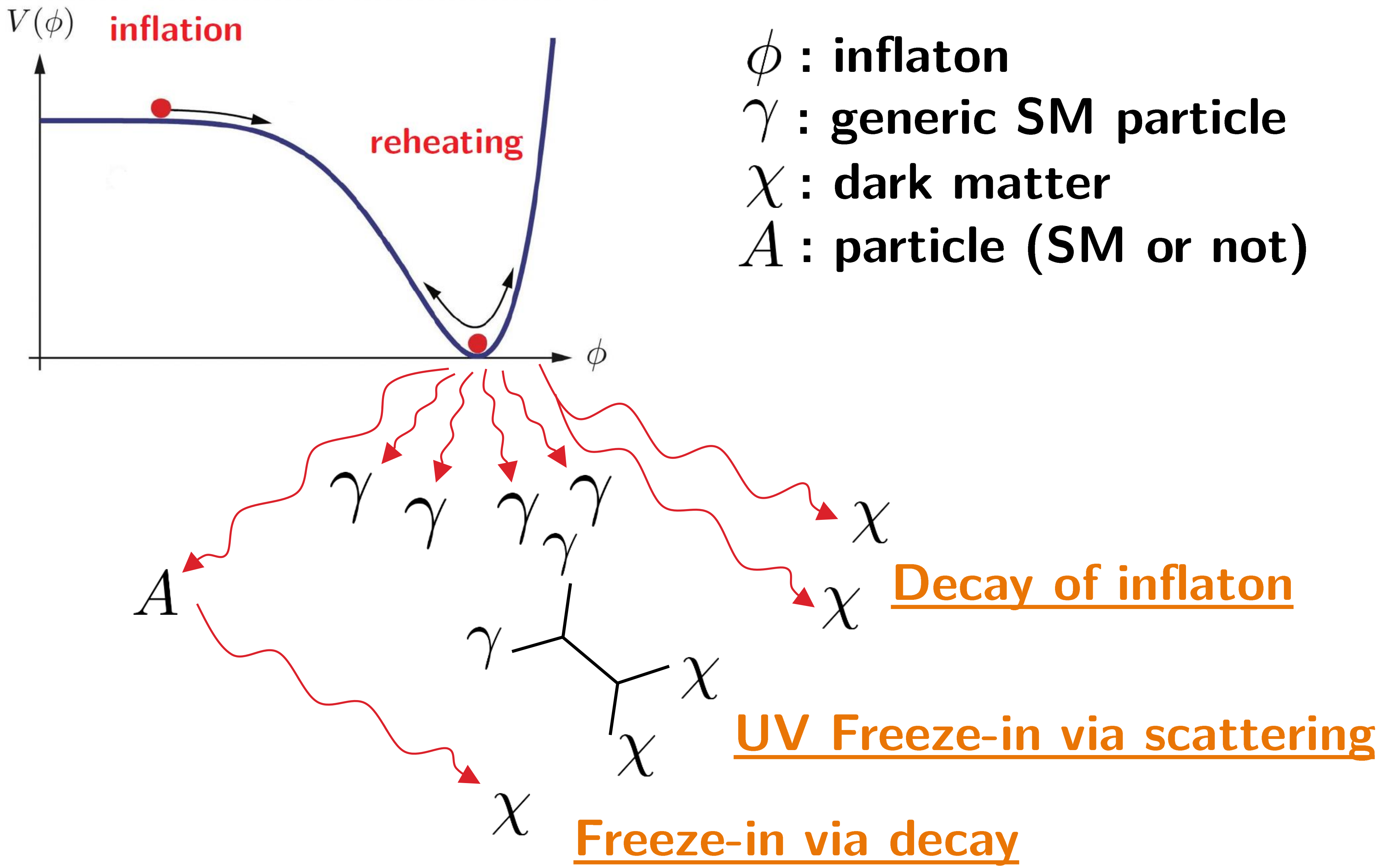


ϕ : inflaton
γ : generic SM particle
χ : dark matter

UV Freeze-in via scattering

Out-of-equilibrium DM production

ϕ : **inflaton**
γ : **generic SM particle**
χ : **dark matter**
A : **particle (SM or not)**



DM Phase space distribution

- Obtain **phase space distribution** by solving **Boltzmann** equation

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial |\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|, t)]$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, f_\chi(p_0, t)$$

number density

$$\rho_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, p_0\, f_\chi(p_0, t)$$

energy density

DM Phase space distribution

- Obtain **phase space distribution** by solving **Boltzmann** equation

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial|\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|, t)]$$

- **Solution** to the Boltzmann equation gives

$$f_\chi(p_0, t) = \int_{t_i}^{t} \mathcal{C}[f_\chi]\left(\frac{a(t)}{a(t')}|\boldsymbol{p}|, t'\right) \mathrm{d}t'$$

- After decoupling, $t > t_{\rm dec}$ DM **production stops**, distribution function only depends on the **comoving momentum**

$$q \equiv \frac{p\,a(t)}{T_\star} \qquad n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\frac{T_\star^3}{a^3}\int \mathrm{d}^3\boldsymbol{q}\,\bar{f}_\chi(q)$$

$T_\star \equiv T_{\rm NCDM}$ in **CLASS** [J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

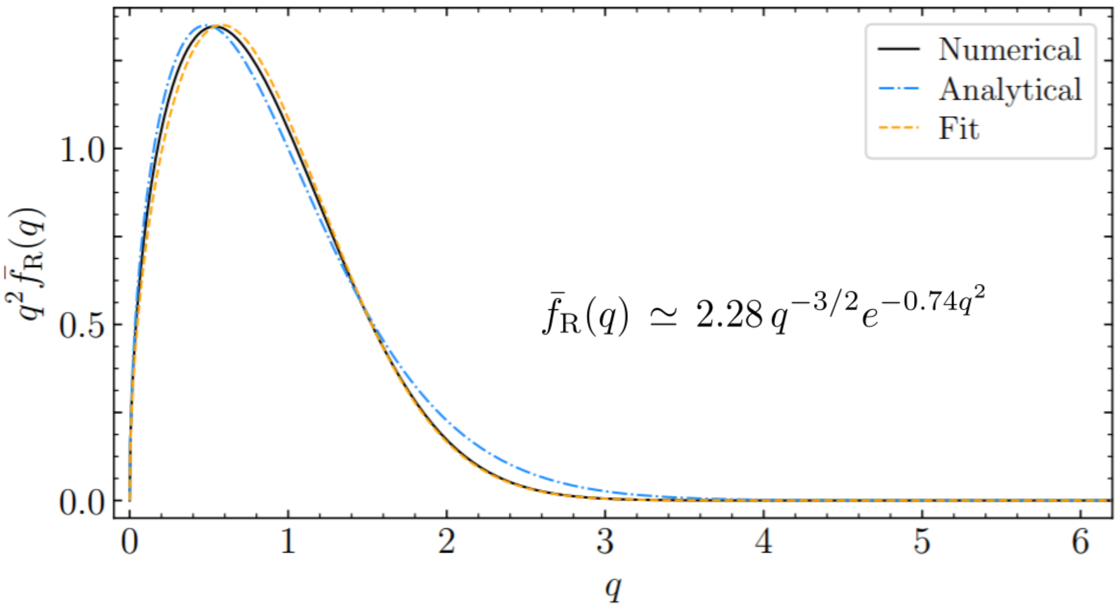
Production from inflaton decay

- Consider **DM** produced from **perturbative** inflaton decay

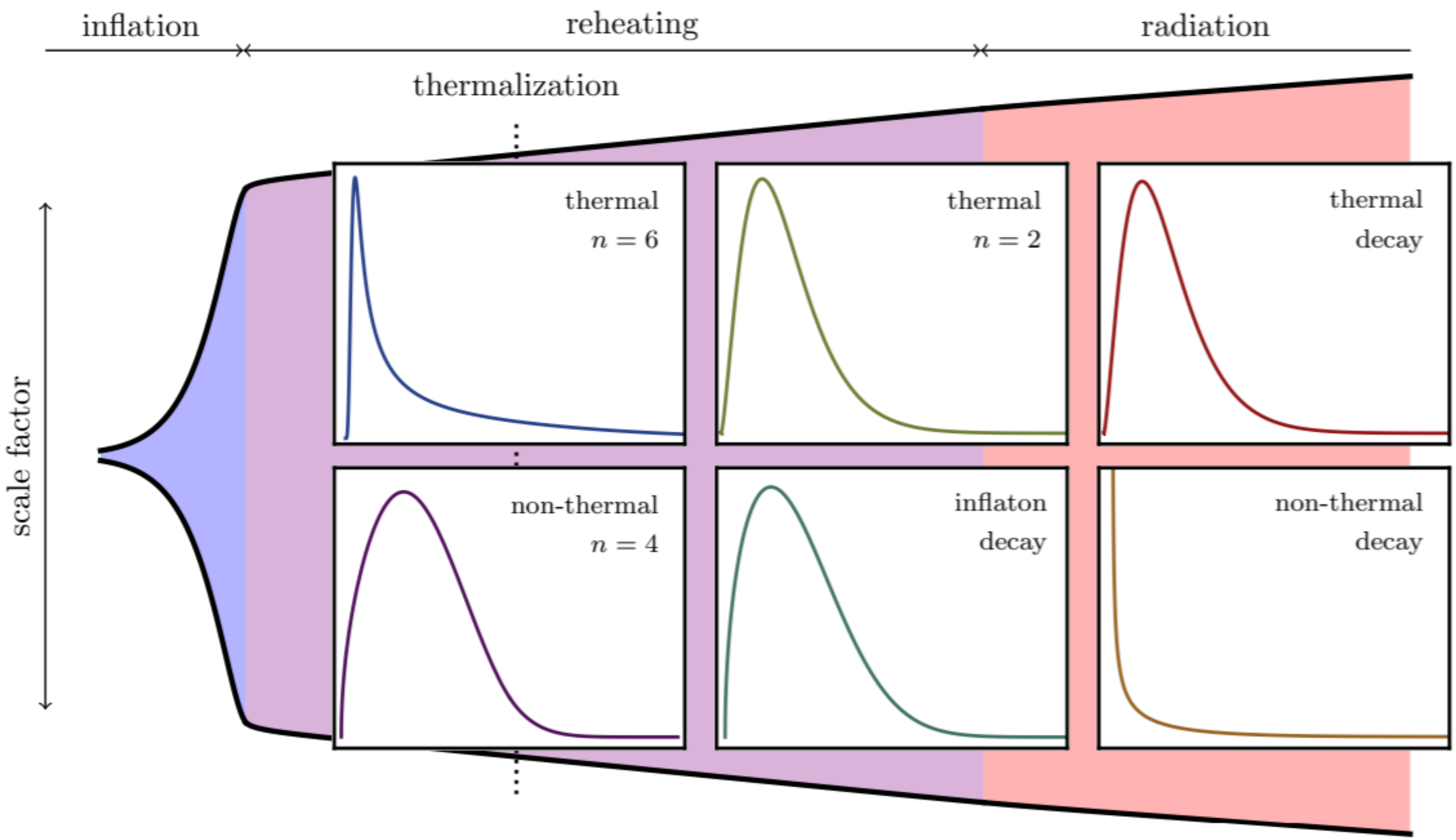
$$\chi \longleftarrow \phi \longrightarrow \chi$$

$$\mathcal{C}[f_\chi(p,t)] = \frac{8\pi^2}{g m_\phi^2}\,\Gamma_\phi\,\mathrm{Br}_\chi\, n_\phi(t)\,\delta(p - m_\phi/2)$$

$$f_\chi(p,t)\;\mathrm{d}^3\boldsymbol{p} = \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi}\left(\frac{T_{\mathrm{reh}}}{m_\phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_\star^3\,\bar{f}_{\mathrm{R}}(q)\;\mathrm{d}^3\boldsymbol{q} \qquad T_\star = \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3}\frac{m_\phi}{2T_{\mathrm{reh}}}\,T_0$$



DM phase space distribution



- Most of the distributions fitted by $f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$

What is the cosmological imprint of out-of-equilibrium dark matter?

Cosmological imprint

Cosmological imprint

- **Cosmological role** of out-of-equilibrium dark matter via

$$\bar{\rho} = 4\pi \left(\frac{T_\star}{a}\right)^4 \int q^2 \epsilon \bar{f}(q)\, \mathrm{d}q$$

energy-density

$$\bar{P} = \frac{4\pi}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q)\, \mathrm{d}q$$

pressure

$$q \equiv \frac{p\, a(t)}{T_\star}$$ **: comoving momentum**

$$\epsilon = \sqrt{q^2 + \left(\frac{m_{\mathrm{DM}}\, a}{T_\star}\right)^2}$$

- Define $w \equiv \bar{P}/\bar{\rho}$ **: equation-of-state parameter**

- In pure $\Lambda\mathrm{CDM}$: $w = 0$ precisely (**Cold = pressureless**)

- But $w \neq 0$! ➡ **Non-Cold Dark Matter** cosmology

Non-Cold Dark Matter $w \neq 0$

- Expanding quantities around **homogenous background**

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = \bar{f}(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)]$$

- In matter domination, matter **overdensities** δ follow

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - \frac{k^2}{k_{\rm FS}^2}\right)\delta = 0 \qquad w \ll 1$$

where $k_{\rm FS}^2(a) = \dfrac{9}{10}\dfrac{\mathcal{H}^2}{w(a)}$ is the **Free-Streaming wavenumber**

$\mathrm{d}\tau \equiv a\,\mathrm{d}t$: **Conformal time** τ

$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**

$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]
[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]
[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]
[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

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where $k_{\rm FS}^2(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w(a)}$ is the **Free-Streaming wavenumber**

- If $w = 0$ all modes grow "**democratically**" : **CDM** limit
  $w \neq 0$ **cutoff in power spectrum** at $k_{\rm H}(a) \equiv \left[\int_0^a \frac{1}{k_{\rm FS}(\tilde{a})} \frac{{\rm d}\tilde{a}}{\tilde{a}}\right]^{-1}$

- **Only w controls the cutoff scale!**

${\rm d}\tau \equiv a\,{\rm d}t$: **Conformal time τ**
$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**
$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

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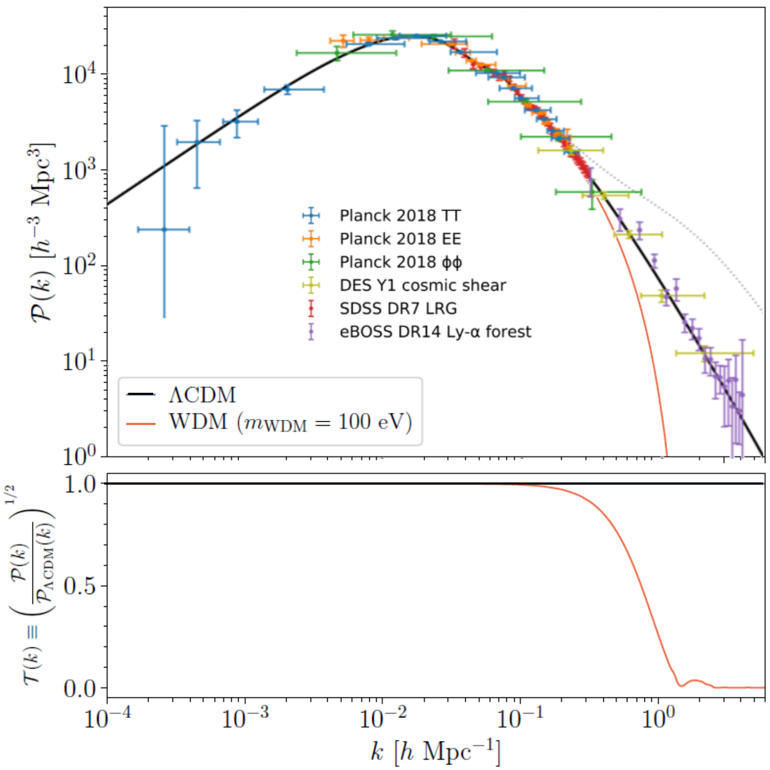
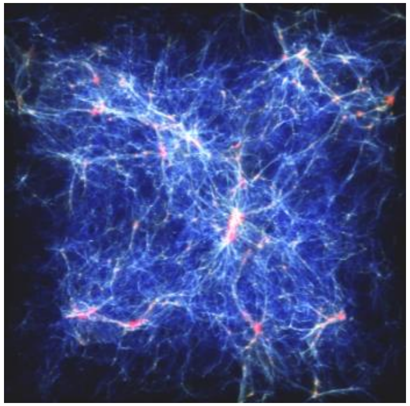
Non-Cold Dark Matter $w \neq 0$

- Lyman-alpha forest constraints Warm Dark Matter (**WDM**)

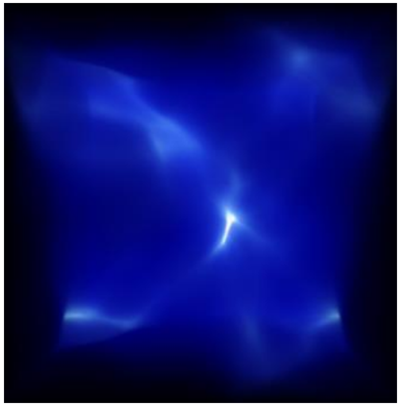
$$\bar{f}_{\rm WDM}(q) = \frac{1}{1+e^{q/T_{\rm WDM}}} \;\Rightarrow\; \Omega_{\rm WDM}h^2 \simeq \left(\frac{m_{\rm WDM}}{94\text{ eV}}\right)\left(\frac{T_{\rm WDM}}{T_\nu}\right)^3 \simeq 0.12$$



ΛCDM



WDM



$m_{\rm WDM} = 100\,\text{eV}$

[J. Baur et al. JCAP 08 (2016) 012]

$$m_{\rm WDM}^{\rm Ly-\alpha} = (1.9 - 5.3)\text{ keV at }95\%\text{ C.L.}$$

$$w_{\rm WDM}(m_{\rm WDM}^{\rm Ly-\alpha}) \sim 10^{-15}a^{-2}$$

[Braur et al. JCAP 08 (2016) 012 – Iršič et al. PRD 96 (2017) 2, 023522
Palanque Delabrouille et al. JCAP 04 (2020) 038 – Viel et al. PRD 88 (2013) 043502
Viel et al. PRD 71 (2005) 063534 – Narayanan et al. ApJ 543 (2000) L103-L106]

How warm is Non-Cold Dark Matter?

- **How to translate Lyman-alpha WDM bounds on any scenario ?**

$$w(m_{\rm DM}) = w_{\rm WDM}(m_{\rm WDM}^{\rm Ly-\alpha})$$

[S. Colombi, S. Dodelson, L. M. Widrow ApJ. 458 (1996) 1 - Kamada, N. Yoshida, K. Kohri, T. Takahashi JCAP 03 (2013) 008 K. J. Bae, R. Jinno, A. Kamada, K. Yanagi JCAP 03 (2020) 042 - A. Kamada & K. Yanagi JCAP 1911 (2019) 029]

$w -$ matching

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3m_{\rm DM}^2}\frac{\langle q^2\rangle}{a^2}$$

$$m_{\rm DM} = m_{\rm WDM}^{\rm Ly-\alpha}\left(\frac{T_\star}{T_{\rm WDM}}\right)\sqrt{\frac{\langle q^2\rangle}{\langle q^2\rangle_{\rm WDM}}}$$

- Compute **2$^{\text{nd}}$ moment of distribution** + **determine** $T_\star$

- If **distribution** can be fitted by $f(q) \propto q^\alpha \exp\left(-\beta\, q^\gamma\right)$

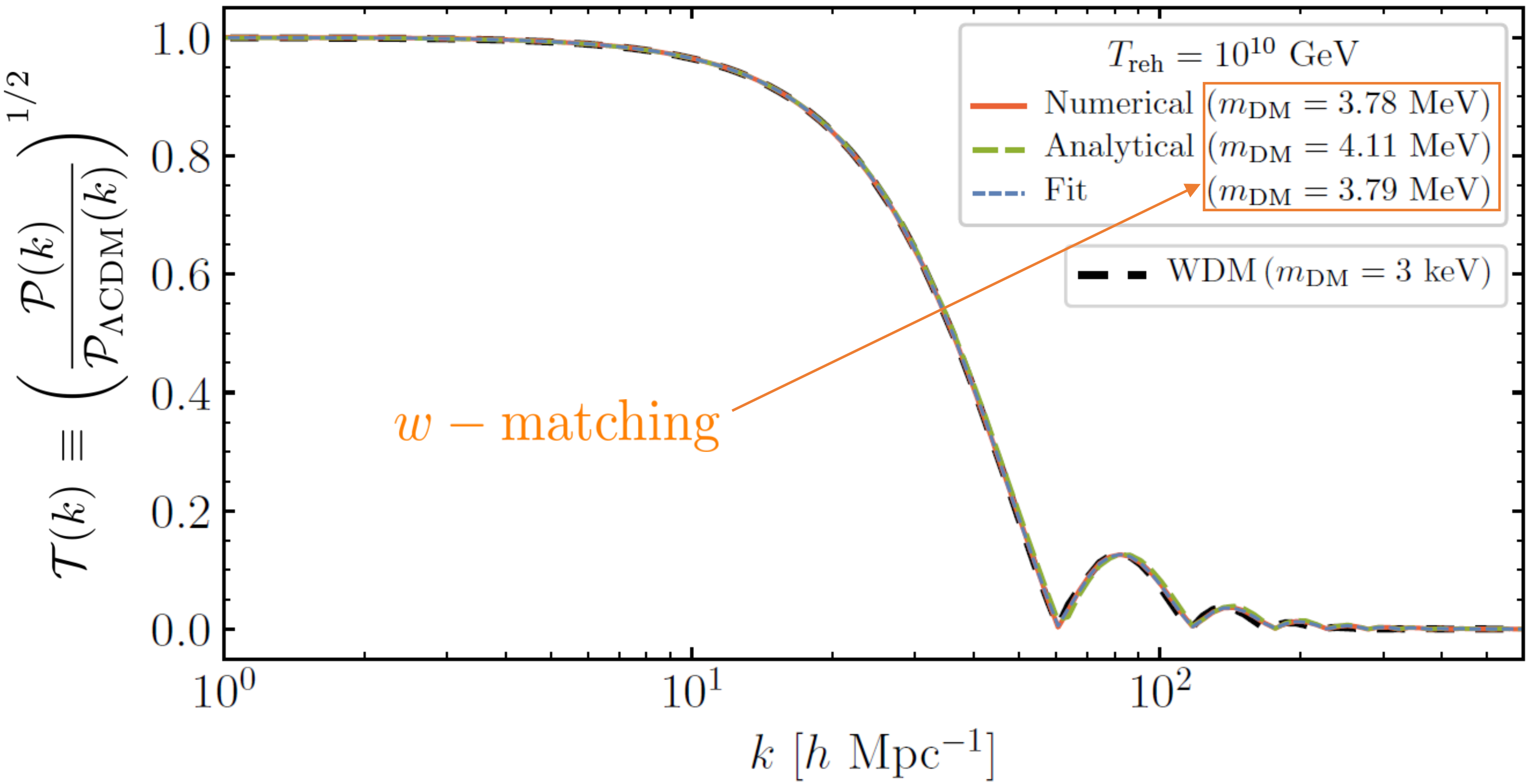
$w -$ matching ⟹

$$m_{\rm DM} \simeq 7.56\ {\rm keV}\left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ {\rm keV}}\right)^{4/3}\left(\frac{\langle p\rangle_0}{T_0}\right)\sqrt{\frac{\Gamma\left(\frac{3+\alpha}{\gamma}\right)\Gamma\left(\frac{5+\alpha}{\gamma}\right)}{\Gamma^2\left(\frac{4+\alpha}{\gamma}\right)}}$$

How warm is Non-Cold Dark Matter?

- Example: **inflaton decay** case computed using **CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- **Excellent agreement** with $w -$ matching for **all distributions**!

Inflaton decay

- **Lyman-alpha** bounds translate into

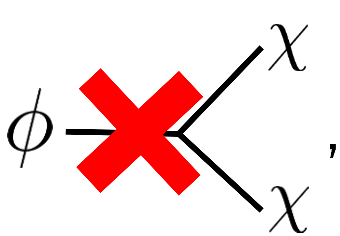
$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \left(\frac{m_\phi}{3\times 10^{13}\ {\rm GeV}}\right) \left(\frac{10^{10}\ {\rm GeV}}{T_{\rm reh}}\right) \begin{cases} 3.78\ {\rm MeV}\,, & {\rm Numerical}\,, \\ 4.11\ {\rm MeV}\,, & {\rm Analytical}\,, \\ 3.79\ {\rm MeV}\,, & {\rm Fit}\,. \end{cases}$$

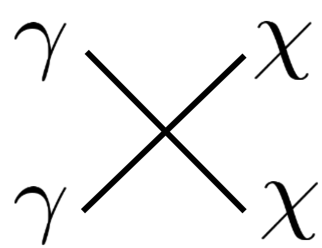
- For **low reheating temperature** $T_{\rm reh} \ll m_\phi$

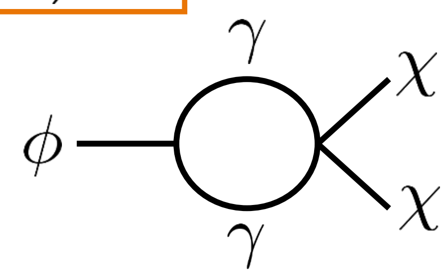
$$\boxed{m_{\rm DM} \gtrsim {\rm EeV}}$$

- **Combining** with **relic density** condition

$$\boxed{{\rm Br}_\chi < 1.5\times 10^{-4} \left(\frac{g_{*s}^{\rm reh}}{106.5}\right)^{1/3} \left(\frac{3\,{\rm keV}}{m_{\rm WDM}^{\rm Ly-\alpha}}\right)^{4/3}}$$

- Even if 

, if 

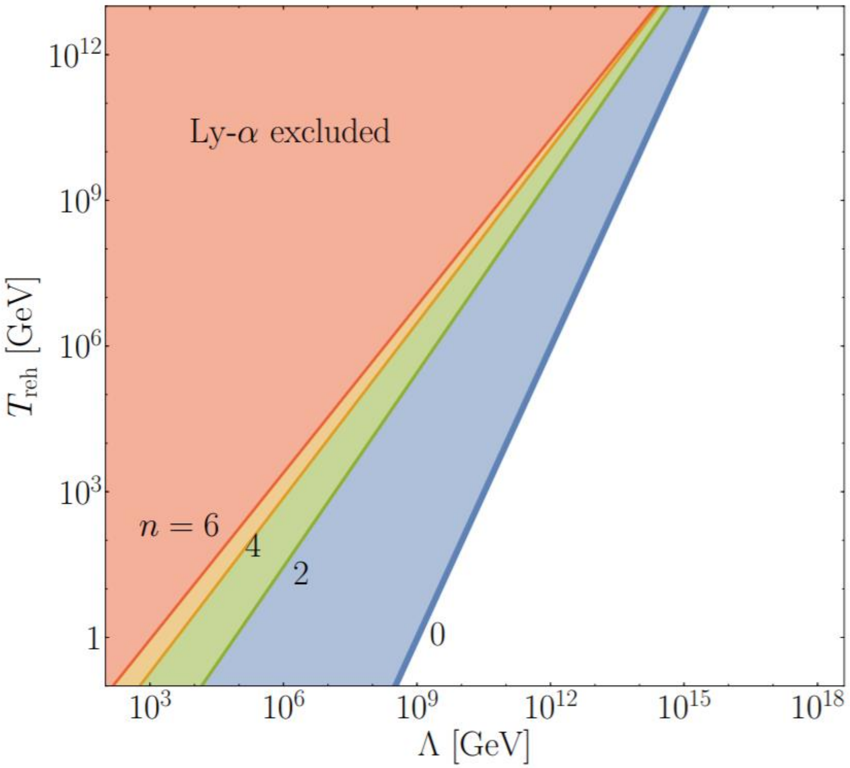
then 

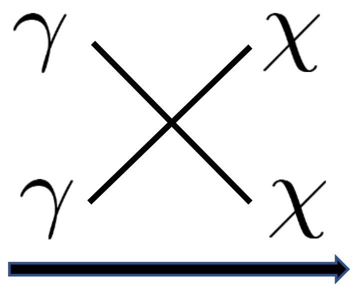
[K. Kaneta, Y. Mambrini & Keith A. Olive Phys.Rev.D 99 (2019) 6, 063508]

UV freeze-in via scattering

$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ {\rm keV}}\right)^{4/3}\left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3}\begin{cases}7.27\ (7.17)\ {\rm keV}\,, & {\rm FF\quad Numerical\ (Fit)}\,, \quad n=0\\ 8.48\ (8.73)\ {\rm keV}\,, & {\rm FF\quad Numerical\ (Fit)}\,, \quad n=2\\ 8.52\ (8.05)\ {\rm keV}\,, & {\rm FF\quad Numerical\ (Fit)}\,, \quad n=4\end{cases}$$

- **Combine** with **relic density** condition





DM production

$$\sigma(s) = \frac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$$

- **Apply to any UV freeze-in model!**

Summary

- **Out-of-equilibrium DM** can be produced **after inflation**
- **Cutoff** in power spectrum can be probed by Lyman-alpha
  Powerful tool to **probe out-of-equilibrium dark matter**
- DM produced from modulus decay, from decay of thermalized particle + much more... **in the paper**! [arXiv:2011.13458]
- **Dark matter is cold**.

Thank you for your attention