



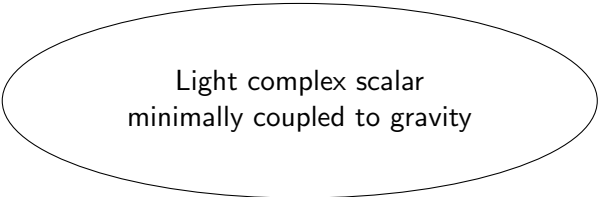
Black Holes as Probes for Ultralight DM

Bruno Bucciotti

2nd June

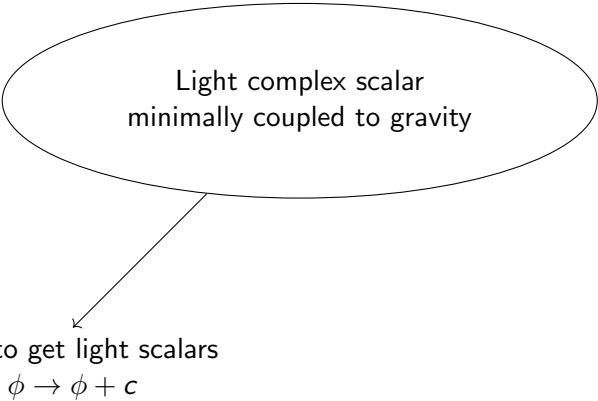
Supervisor: Enrico Trincherini

$$10^{-21} \text{ eV} \lesssim \mu \lesssim 10 \text{ eV}$$



Light complex scalar
minimally coupled to gravity

$$10^{-21} \text{ eV} \lesssim \mu \lesssim 10 \text{ eV}$$



Light complex scalar
minimally coupled to gravity

Easy to get light scalars

$$\phi \rightarrow \phi + c$$

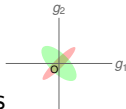
$$10^{-21} \text{ eV} \lesssim \mu \lesssim 10 \text{ eV}$$

Light complex scalar
minimally coupled to gravity

Easy to get light scalars

$$\phi \rightarrow \phi + c$$

Assume no interactions



$$10^{-21} \text{ eV} \lesssim \mu \lesssim 10 \text{ eV}$$

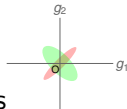
Light complex scalar
minimally coupled to gravity

Easy to get light scalars

$$\phi \rightarrow \phi + c$$

Assume no interactions

Interplay with black holes



Goals

- ▶ Analytically understand the zoo of solutions
- ▶ Check numerical results
- ▶ Extend previous work
- ▶ Emphasize horizon effects

ϕ

(classical EoM)

What is the field profile?

Dependence on μ ?

Relation $|\phi|_{r \rightarrow \infty}$ to $|\phi|_{r \rightarrow r_{BH}}$?

$$\square\phi - \mu^2\phi = 0$$

(classical EoM)

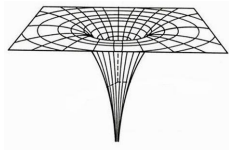
What is the field profile?

Dependence on μ ?

Relation $|\phi|_{r \rightarrow \infty}$ to $|\phi|_{r \rightarrow r_{BH}}$?

$$\square\phi - \mu^2\phi = 0$$

(classical EoM)



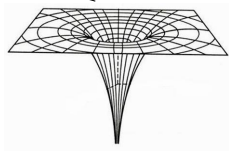
What is the field profile?

Dependence on μ ?

Relation $|\phi|_{r \rightarrow \infty}$ to $|\phi|_{r \rightarrow r_{BH}}$?

$$\square\phi - \mu^2\phi = 0$$

(classical EoM)



- ▶ BH domination at first, no self gravity
 - ▶ Nonrotating black hole
- Schwarzschild metric

What is the field profile?

Dependence on μ ?

Relation $|\phi|_{r \rightarrow \infty}$ to $|\phi|_{r \rightarrow r_{BH}}$?

Hairy solutions

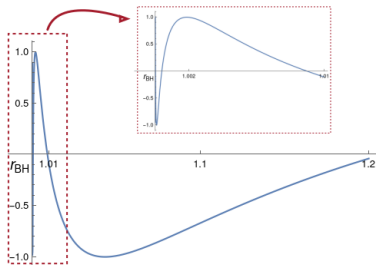
$$\phi \rightarrow e^{-i\omega t} Y_{l,m} \phi(r), \quad \omega = \mu, \quad V_l = r_{BH} \mu^2 r^3 - l(l+1)r(r-r_{BH})$$

$$r(r-r_{BH}) \frac{d}{dr} \left(r(r-r_{BH}) \frac{d\phi}{dr} \right) + V_l(r) \phi = 0$$

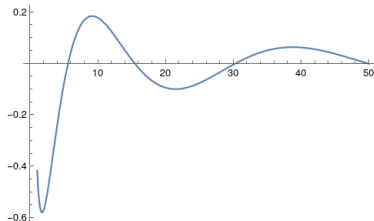
Hairy solutions

$$\phi \rightarrow e^{-i\omega t} Y_{l,m} \phi(r), \quad \omega = \mu, \quad V_l = r_{BH} \mu^2 r^3 - l(l+1)r(r-r_{BH})$$

$$r(r-r_{BH}) \frac{d}{dr} \left(r(r-r_{BH}) \frac{d\phi}{dr} \right) + V_l(r) \phi = 0$$



Near horizon limit
(numerically hard)



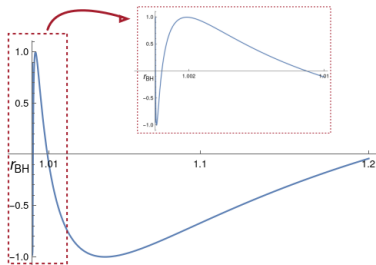
Far field

Local WKB

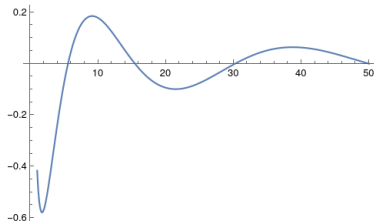
Hairy solutions

$$\phi \rightarrow e^{-i\omega t} Y_{l,m} \phi(r), \quad \omega = \mu, \quad V_l = r_{BH} \mu^2 r^3 - l(l+1)r(r-r_{BH})$$

$$r(r-r_{BH}) \frac{d}{dr} \left(r(r-r_{BH}) \frac{d\phi}{dr} \right) + V_l(r)\phi = 0$$



Near horizon limit
(numerically hard)



Far field

Local WKB

Can they be 'glued'?

$l = 0$ in [Hui '19]

Global WKB

$$r_{SgrA^*} \simeq 10^7 \text{ km} \simeq 10^{17} \text{ eV}^{-1}$$

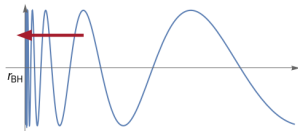
$$(r - r_{BH})^{-i\mu r_{BH}} \longleftrightarrow \frac{1}{r^{3/4}} e^{-2i\mu\sqrt{r_{BH}r}}$$

$$|\phi_l| \propto V_l^{-1/4}(r)$$

l	0	1	2
$\mu r_{BH} \gtrsim$	0.3	0.7	1.2

Impose causal infalling b.c. at horizon

(Nontrivial implications:
Love numbers, no hair thms.)



Global WKB

$$r_{SgrA^*} \simeq 10^7 \text{ km} \simeq 10^{17} \text{ eV}^{-1}$$

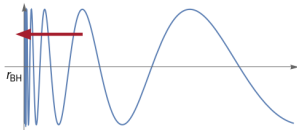
$$(r - r_{BH})^{-i\mu r_{BH}} \longleftrightarrow \frac{1}{r^{3/4}} e^{-2i\mu\sqrt{r_{BH}r}}$$

$$|\phi_l| \propto V_l^{-1/4}(r)$$

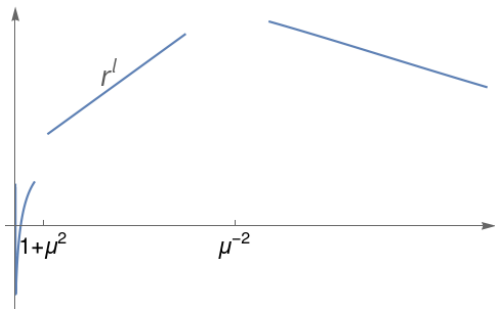
l	0	1	2
$\mu r_{BH} \gtrsim$	0.3	0.7	1.2

Impose causal infalling b.c. at horizon

(Nontrivial implications:
Love numbers, no hair thms.)



Small μr_{BH} ?



$$l = 1$$

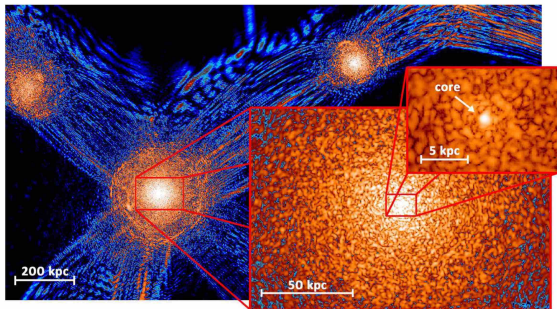
$$\mathcal{O}(\mu r_{BH})^{\frac{3}{2}+2l} \sim (\mu r_{BH})^{\frac{3}{2}+2l} r^l \sim \frac{1}{r^{3/4}} \cos(2\mu\sqrt{r_{BH}r})$$

$\mathcal{O}(1)$ variations in the density profile

- ✓ Solution for all distances
- ✓ Analytic control (causality)
- ✓ μ dependence
- ✓ $|\phi_\infty/\phi_{hor.}|$

- ✓ Solution for all distances
- ✓ Analytic control (causality)
- ✓ μ dependence
- ✓ $|\phi_\infty/\phi_{hor.}|$

$\omega < \mu$ allowed, but needs $Im[\omega] < 0 \rightarrow e^{-Im[\omega]t}$ decay!



Soliton (bound state). [Schive et al. 2014]

Solitons

$$M_s \simeq 10^9 M_\odot \propto M_{halo}^{1/3} (?)$$

easier

harder

BH domination

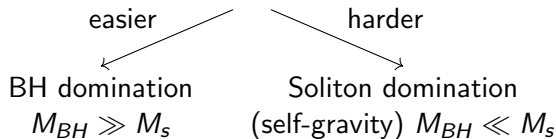
$$M_{BH} \gg M_s$$

Soliton domination

(self-gravity) $M_{BH} \ll M_s$

Solitons

$$M_s \simeq 10^9 M_\odot \propto M_{halo}^{1/3} (?)$$



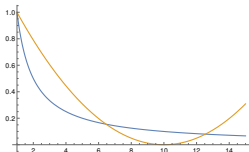
Aim to

- ▶ Soliton domination
- ▶ Solution at all distances
- ▶ Causal boundary conditions
- ▶ GR gravity close to the horizon
- ▶ No non-relativistic approximation



$$\alpha \phi_c + (1 - \alpha) \phi_{n.c}$$

- ▶ Large μr_{BH} : b.c. important at large distances
- ▶ Small μr_{BH} : b.c. important only $\sim r_{BH}$ away, but maybe this is already outside the sphere of influence of the BH!



$$l = 0^1, \quad \nabla^2 \Phi_N = -4\pi G\rho$$

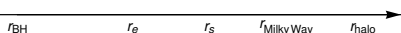
$$V \rightarrow V - 2\mu^2 r^4 \Phi_N + (\omega^2 - \mu^2)r^4$$

¹Instabilities at $l > 0$ (?) [Dmitriev '21]

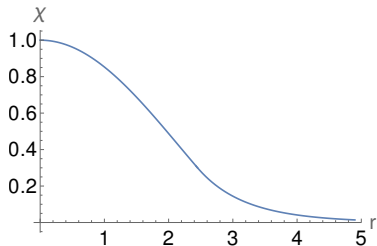
Sketch: given Φ_N , compute ϕ in WKB \rightarrow compute ρ, Φ_N

$$r_{BH} \simeq 10^7 \text{ km}, \quad r_e \simeq 10^{13} \text{ km} \simeq 1 \text{ pc}$$

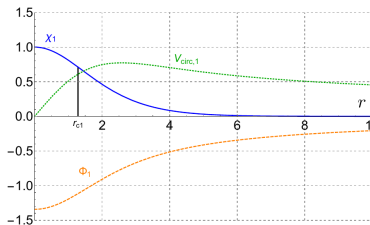
$$r_s \simeq 10^{15} \text{ km}, \quad r_{M.W.} \simeq 10^{17} \text{ km}$$



$$r_s = \frac{1}{\mu^2 GM_s}, \quad r_{n.l.} = r_s \frac{M_{BH}}{M_s}, \quad \Delta M_s / M_s \simeq 10^{-8} \text{ in } 10^{10} \text{ y} \propto \left(\frac{r_{BH}}{r_s}\right)^2 \frac{1}{r_s}$$



Analytic solution



Bar, Blas, Blum, Sibirakov

Stability is crucial! It selects small μr_{BH}

$$GM_s \ll r_s \rightarrow GM_s \ll \frac{1}{\mu^2 GM_s} \rightarrow \mu GM_s \ll 1$$
$$\rightarrow \mu r_{BH} \ll \frac{M_{BH}}{M_s} \ll 1$$

The hierarchy of scales makes boundary conditions unimportant at large distances

$$r_s \gg GM_s \rightarrow r_{n.l.} \gg GM_s \frac{M_{BH}}{M_s} = r_{BH}$$

GR corrections far away are small or the soliton is unstable.

For $M_{BH} > M_s$ the known result $GM_{BH} r_s \mu^2 \simeq \mathcal{O}(1)$ gives

$$\mu r_{BH} \ll \sqrt{\frac{M_{BH}}{M_s}}$$

Conclusions

Hairy black hole:

- ✓ evade no-hair theorems
- ✓ $\rho(r)$ for spinning DM
- ⚠ boundary conditions significant when $\mu r_{BH} \gtrsim \mathcal{O}(1)$

Soliton:

- ✓ $\rho(r)$, with self-gravity
- ✓ b.c unimportant in soliton domination
- ✓ b.c unimportant in BH domination, small μr_{BH}
- ⚠ b.c. significant when $M_{BH} > M_s$ and $\mu r_{BH} \gtrsim \mathcal{O}(1)$

For $SgrA^*$: $M_s \lesssim 10^7 M_\odot$, $\mu \gtrsim 10^{-17} \text{eV}$

Future directions

Include self interactions. Consider more scalars