# Probing dark matter with strong lensing and the line of sight

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Lucia Marchetti, and many others

# An overview:

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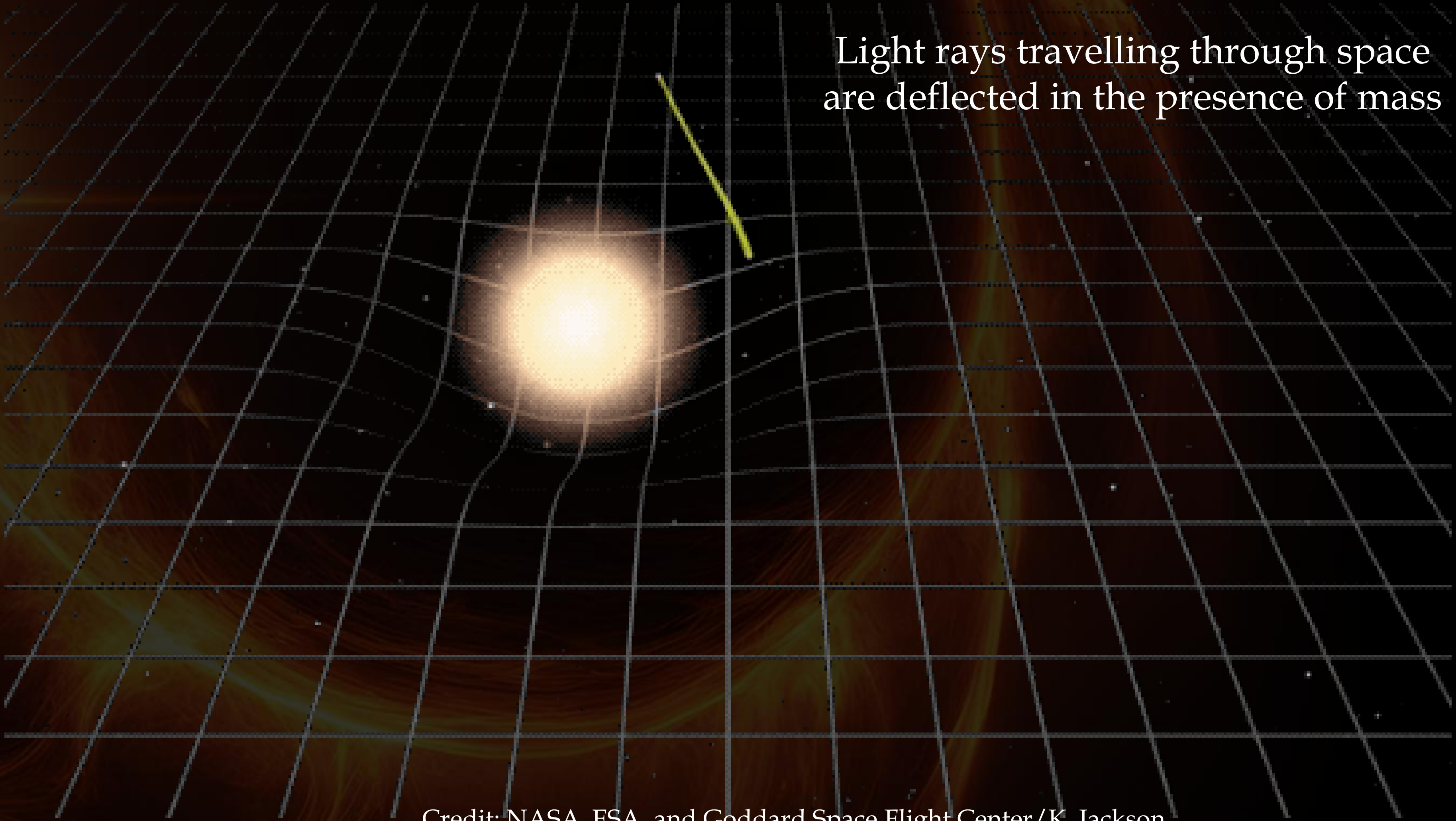
1. Gravitational lensing is an excellent probe of dark matter
2. Strong gravitational lensing is affected by matter along the line of sight of the lens system (the "weak lensing of strong lensing")
3. These effects offer novel constraints on the matter power spectrum

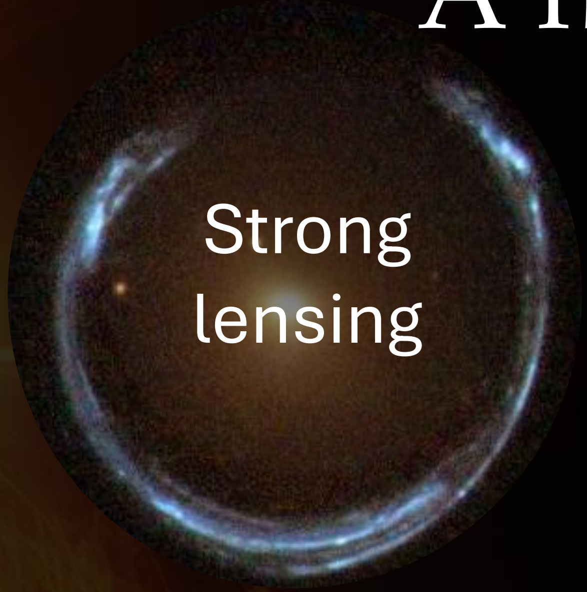
# Gravitational lensing

Light rays travelling through space are deflected in the presence of mass



Credit: NASA, ESA, and Goddard Space Flight Center/K. Jackson

# A historical division





Strong, non-linear distortions by a galaxy or galaxy cluster, often resulting in multiple images

Small distortions to magnifications and shapes – an integrated effect along the line of sight

# Lensing and dark matter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Galaxy dark matter content by comparing light and mass profiles

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Dark matter substructures from perturbations to Einstein rings

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The matter power spectrum

# Lensing and dark matter


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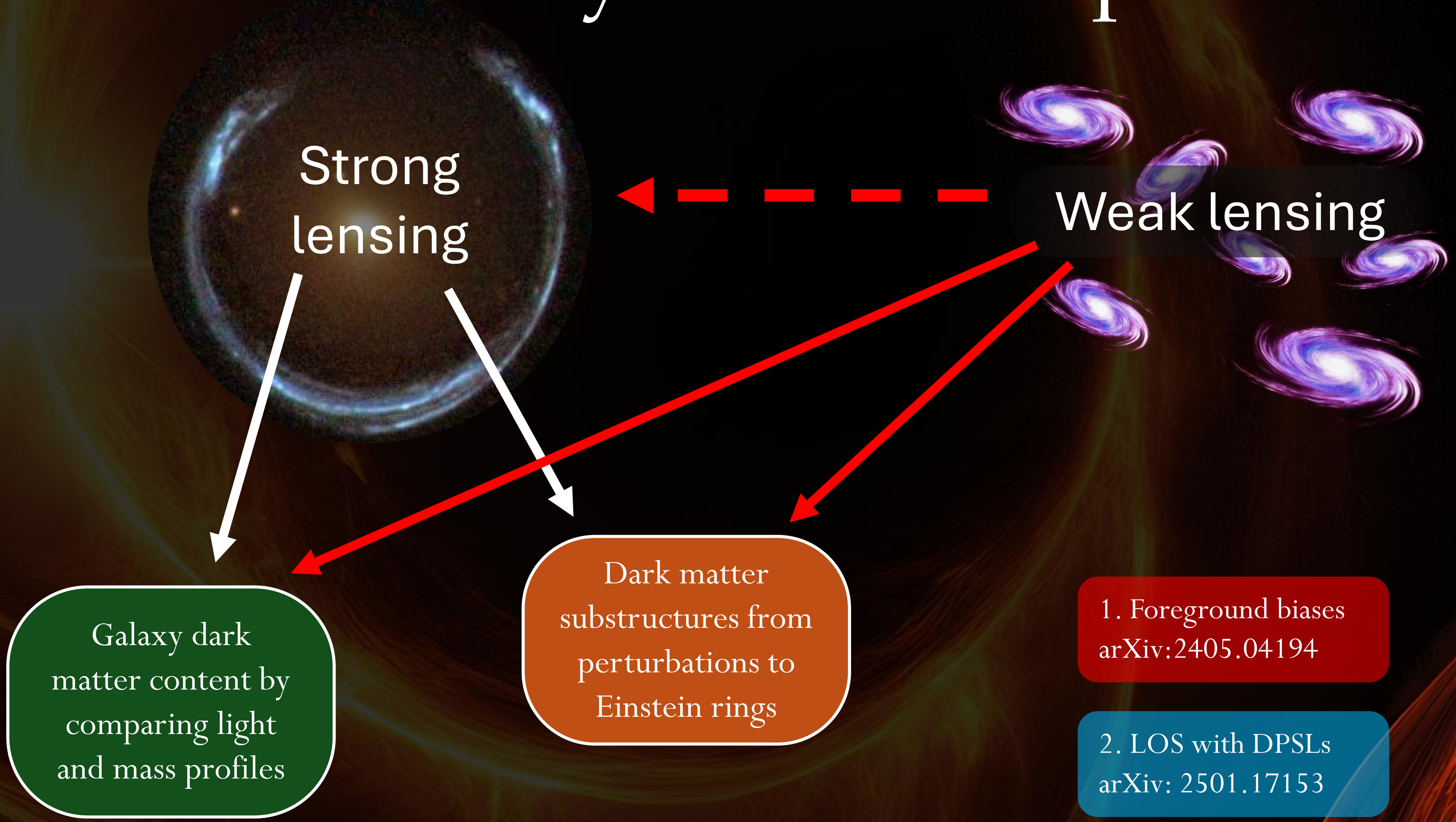
Dark matter substructures from perturbations to Einstein rings


$\Omega_m, \sigma_8$

The matter power spectrum

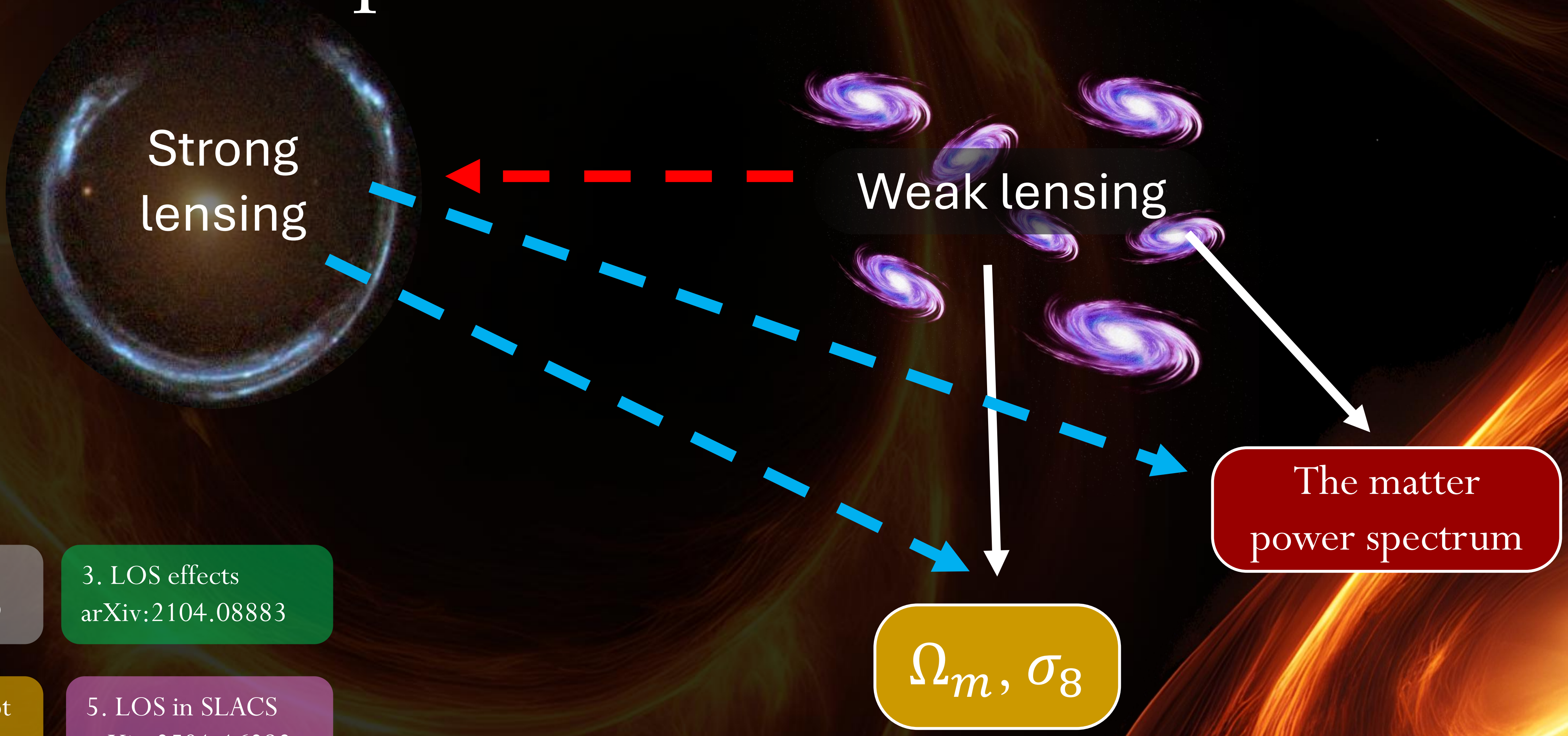
# But they're not independent!




1. Foreground biases
arXiv:2405.04194

2. LOS with DPSLs
arXiv: 2501.17153

# Perhaps this could be useful?




LOS inference
arXiv:1610.01599

3. LOS effects
arXiv:2104.08883

4. Proof of concept
arXiv:2210.07210

5. LOS in SLACS
arXiv:2501.16292

# Weak lensing of galaxies

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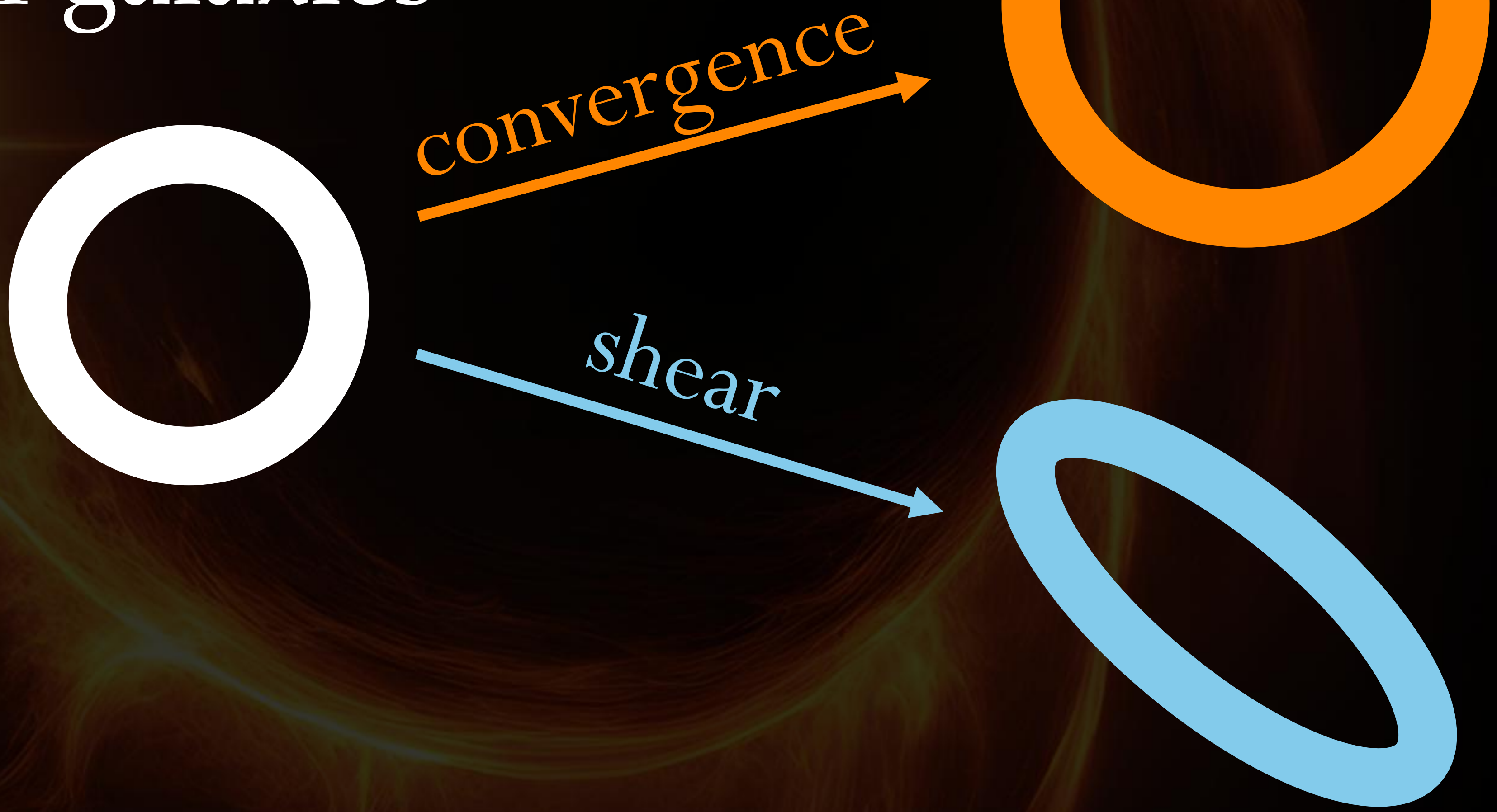

# Strong lensing

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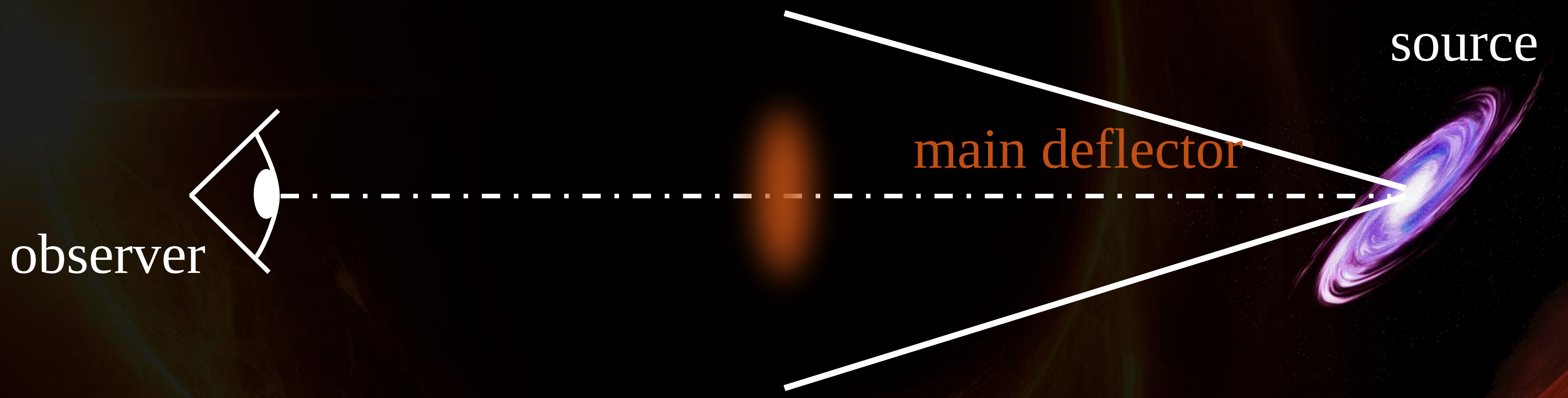

# Strong lensing

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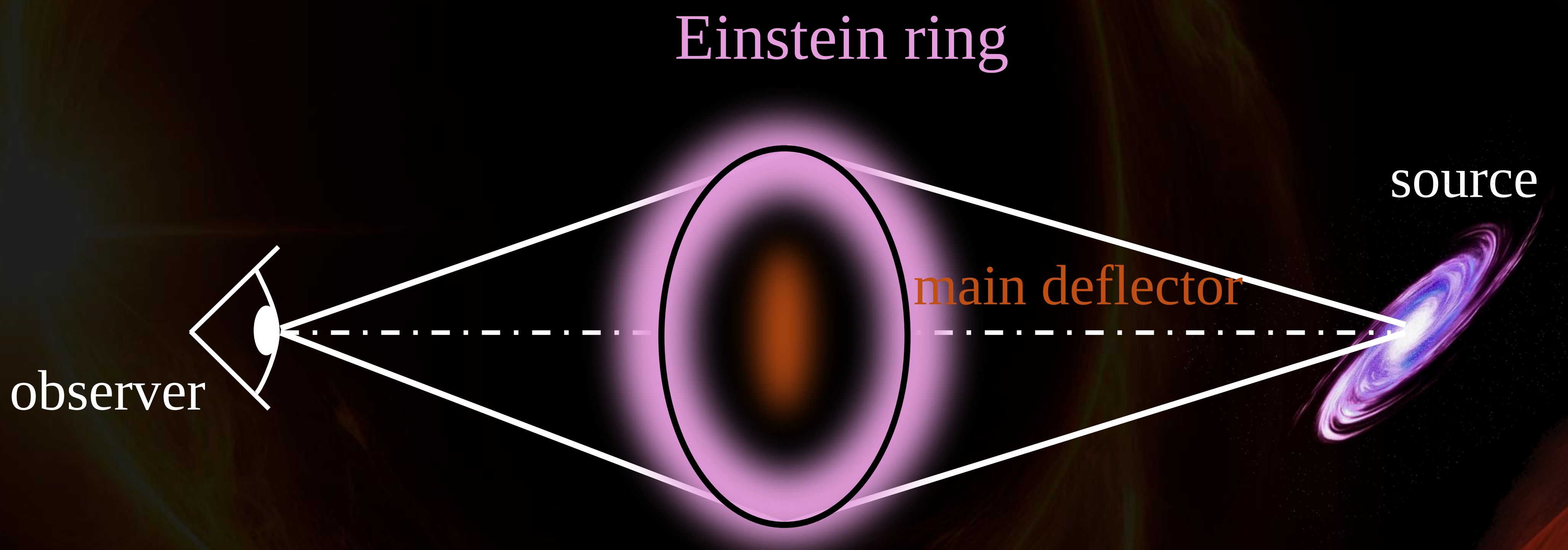
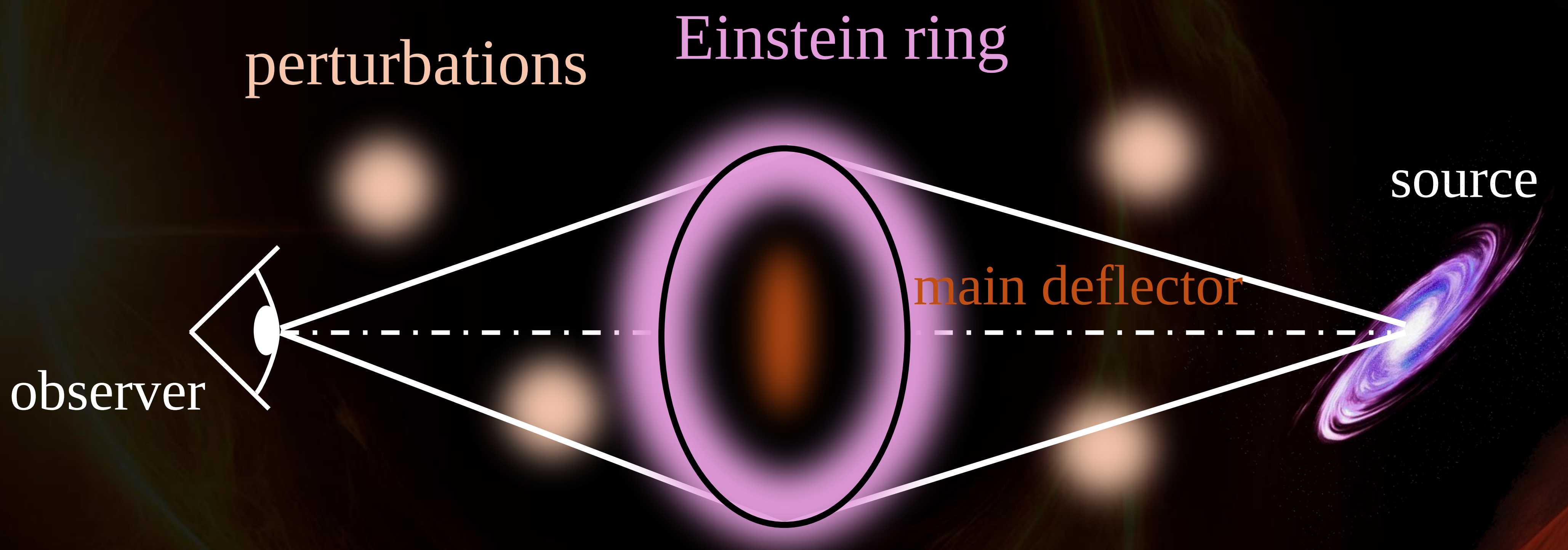


# Weak lensing of strong lensing



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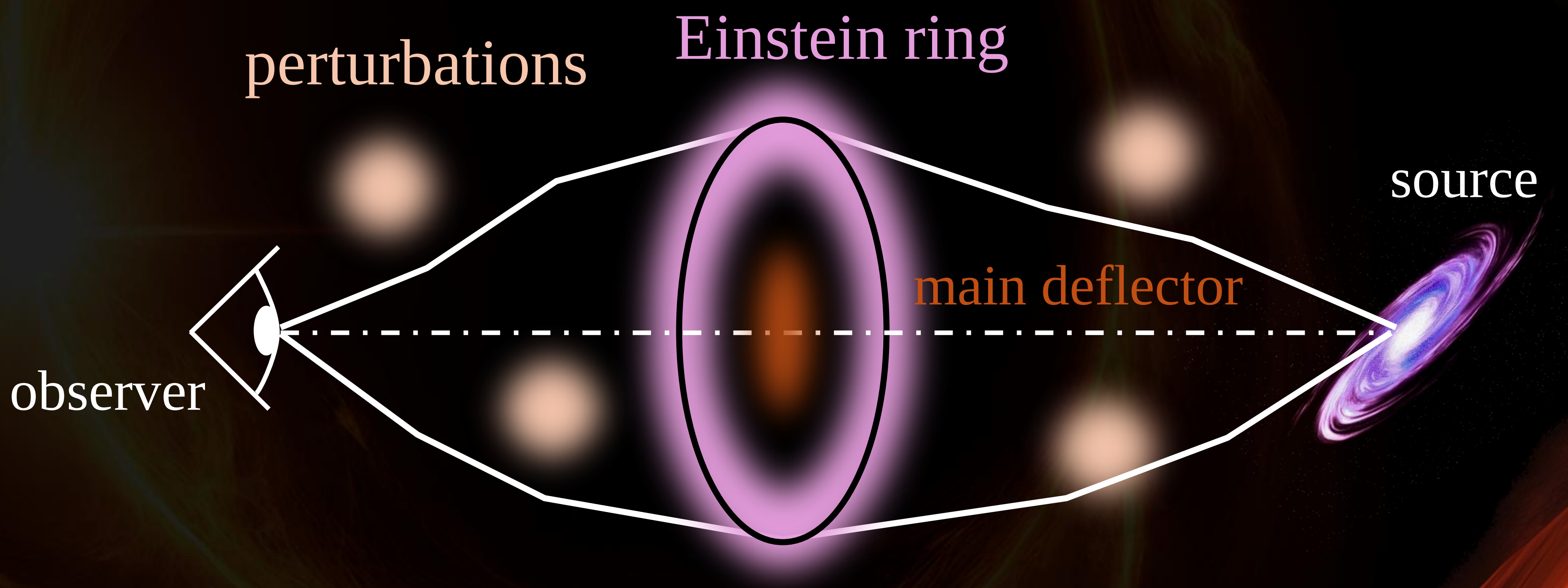
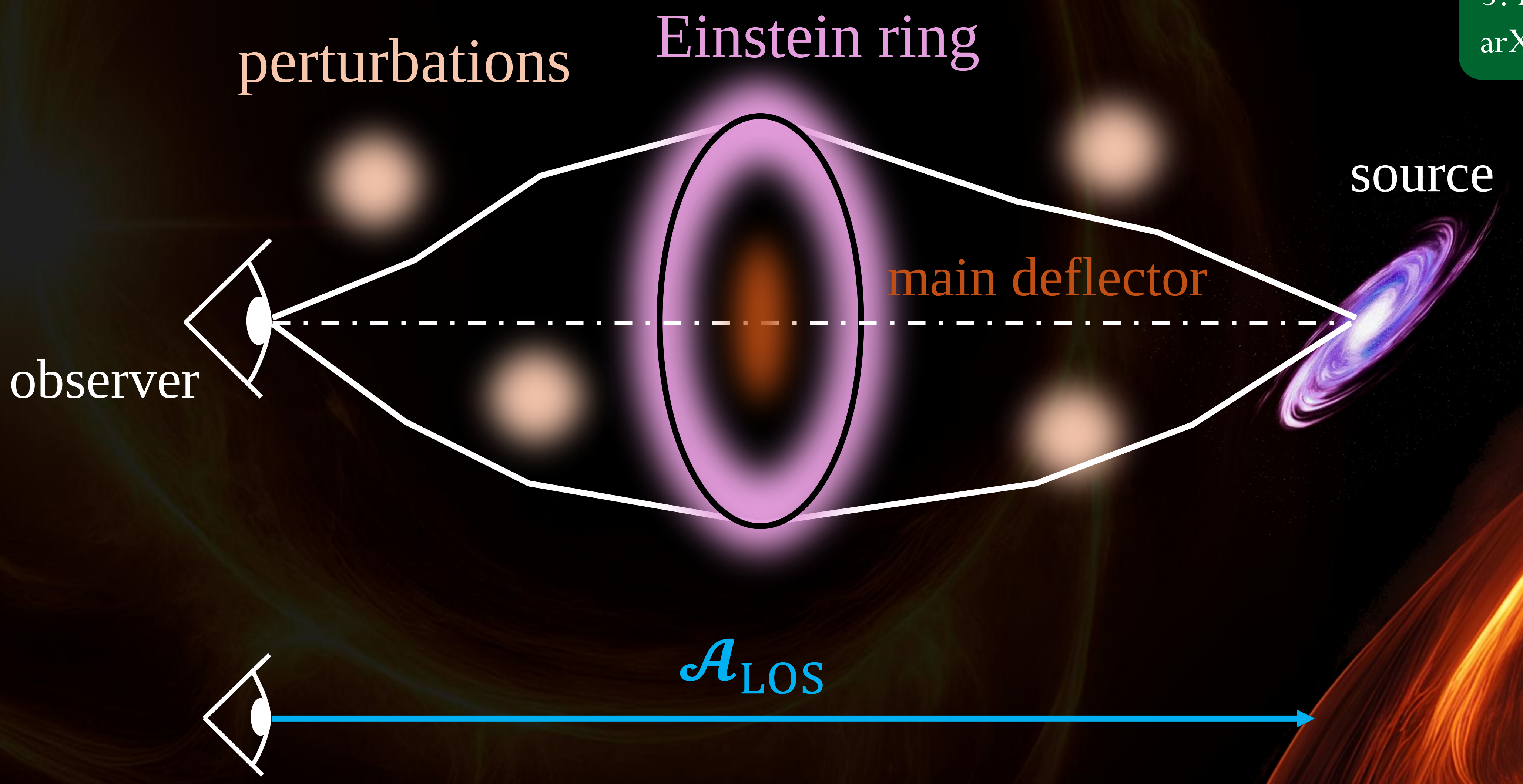
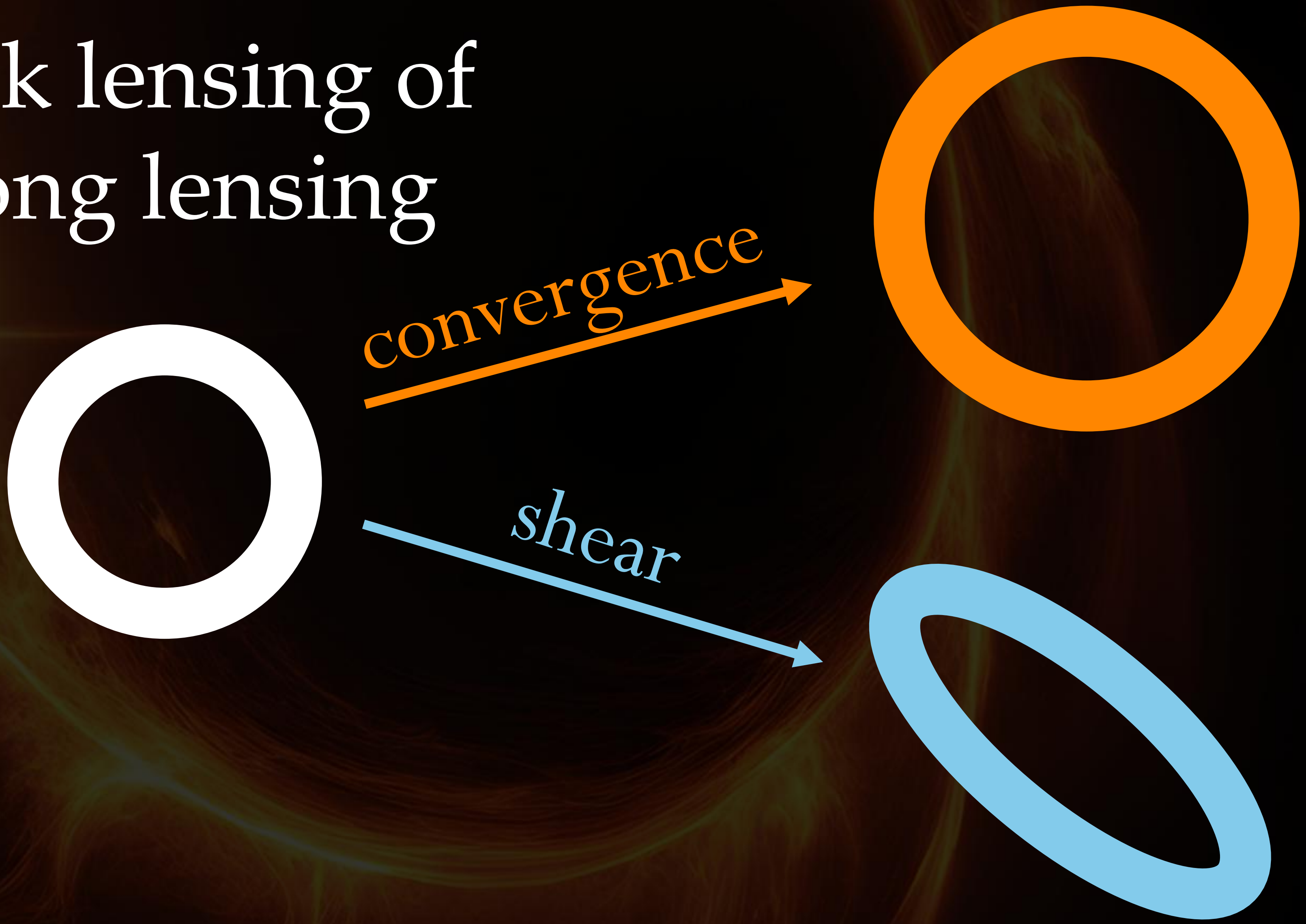


# Weak lensing of strong lensing

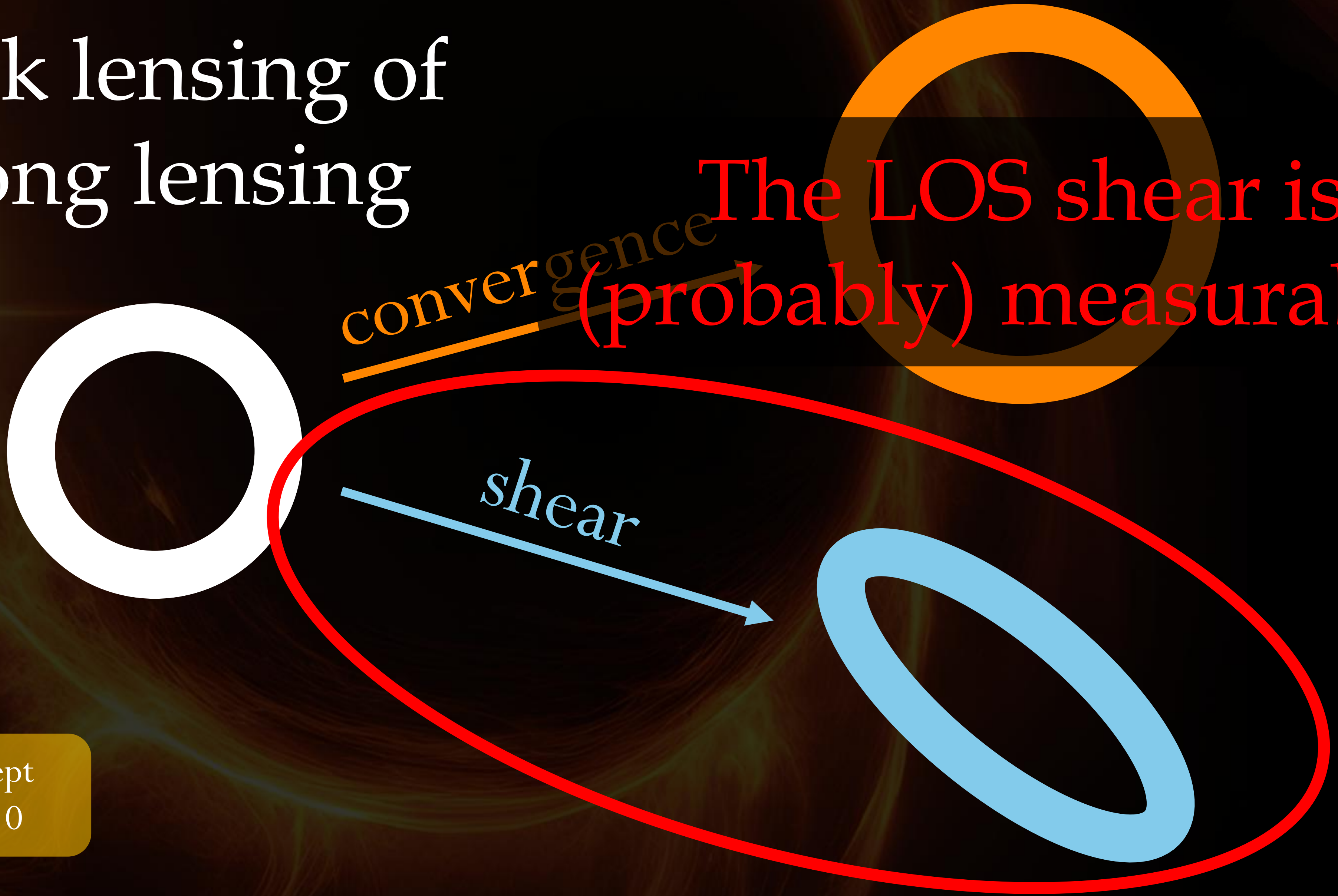
3. LOS effects
arXiv:2104.08883



# Weak lensing of strong lensing



# Weak lensing of strong lensing




The LOS shear is (probably) measurable!

4. Proof of concept
arXiv:2210.07210

# Why is this interesting?

$$\gamma_{\mathrm{LOS}} \sim -\Omega_m \int_0^{\chi_\mathrm{s}} \mathrm{d}\chi \int_{\mathbb{R}^2} \mathrm{d}^2\boldsymbol{x}\ \delta$$

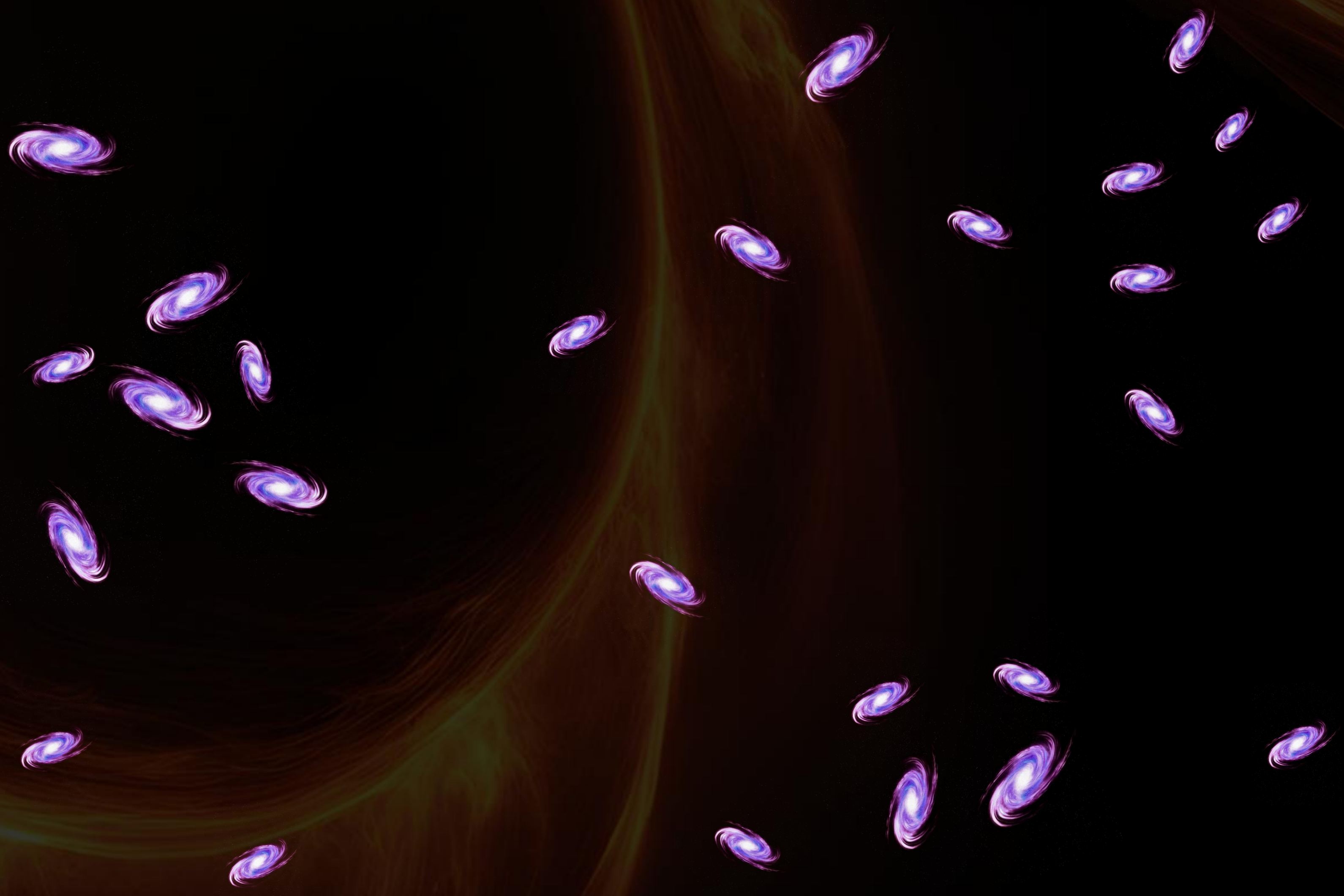
line-of-sight shear

matter density parameter

density contrast

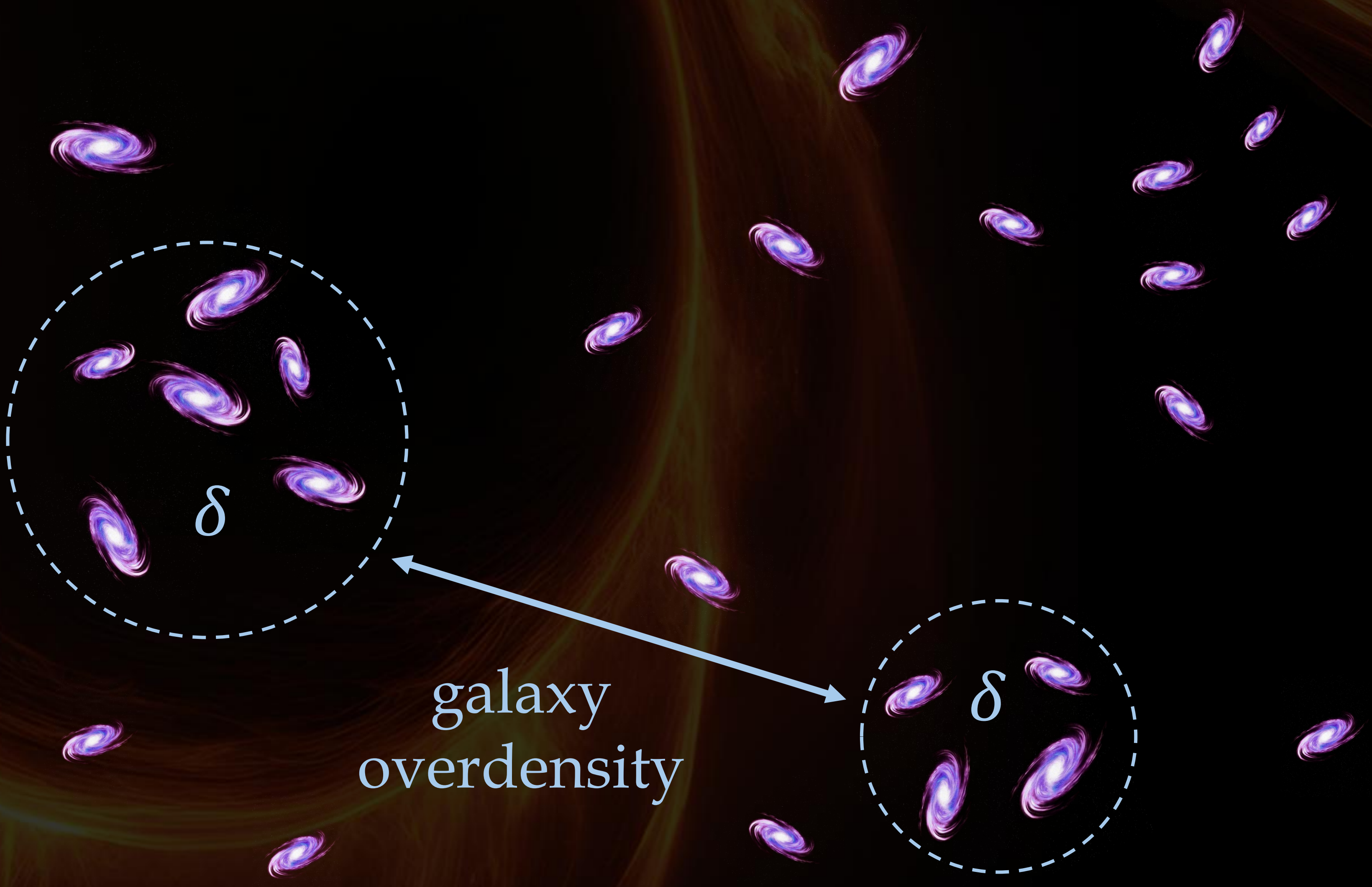
$\gamma_{\mathrm{LOS}}$ depends on the matter power spectrum via an integral of $\delta$ along the line of sight

# The 3x2pt correlation scheme



# The 3x2pt correlation scheme

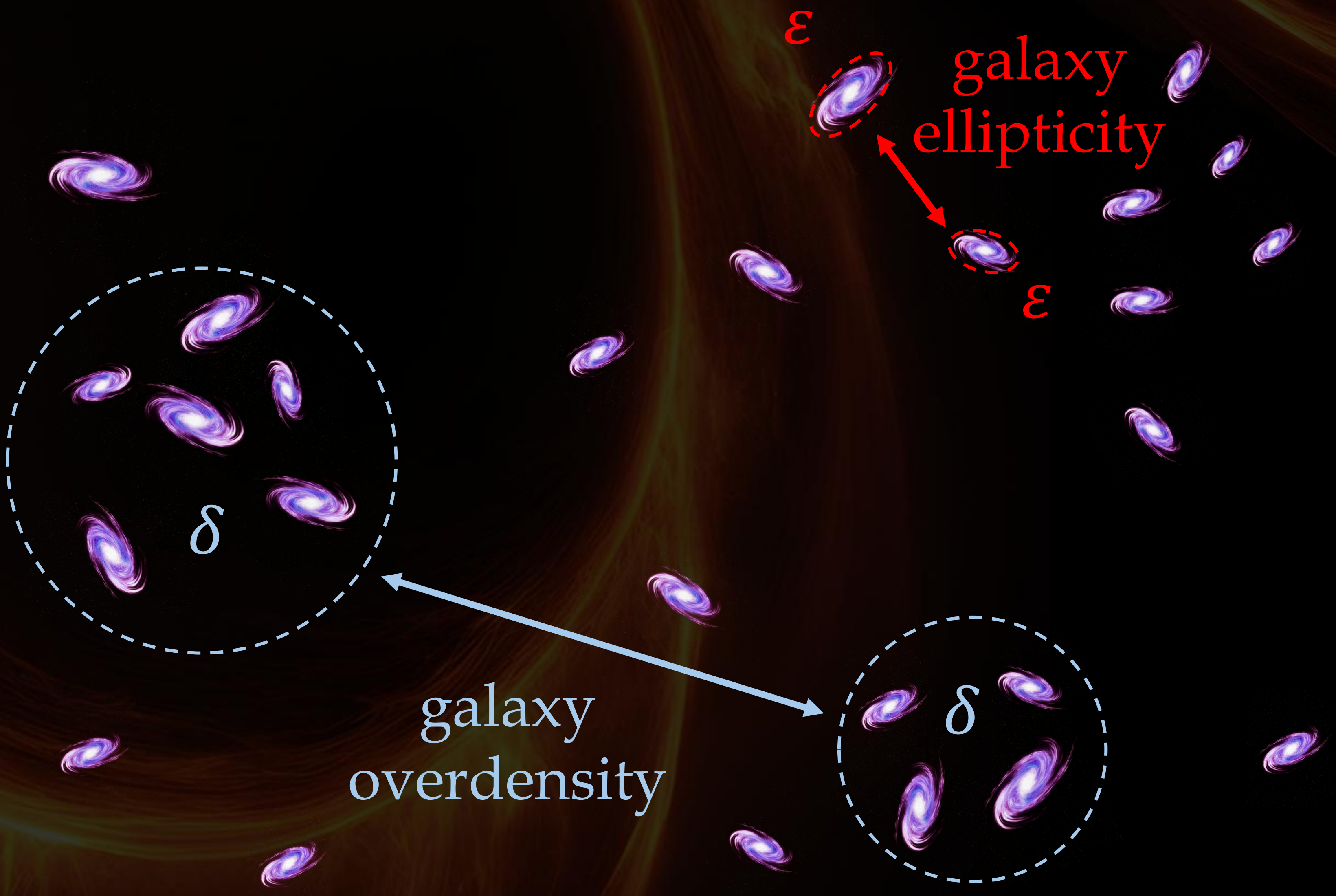
$\langle \delta \times \delta \rangle$



# The 3x2pt correlation scheme

$\langle \delta \times \delta \rangle$

$\langle \varepsilon \times \varepsilon \rangle$



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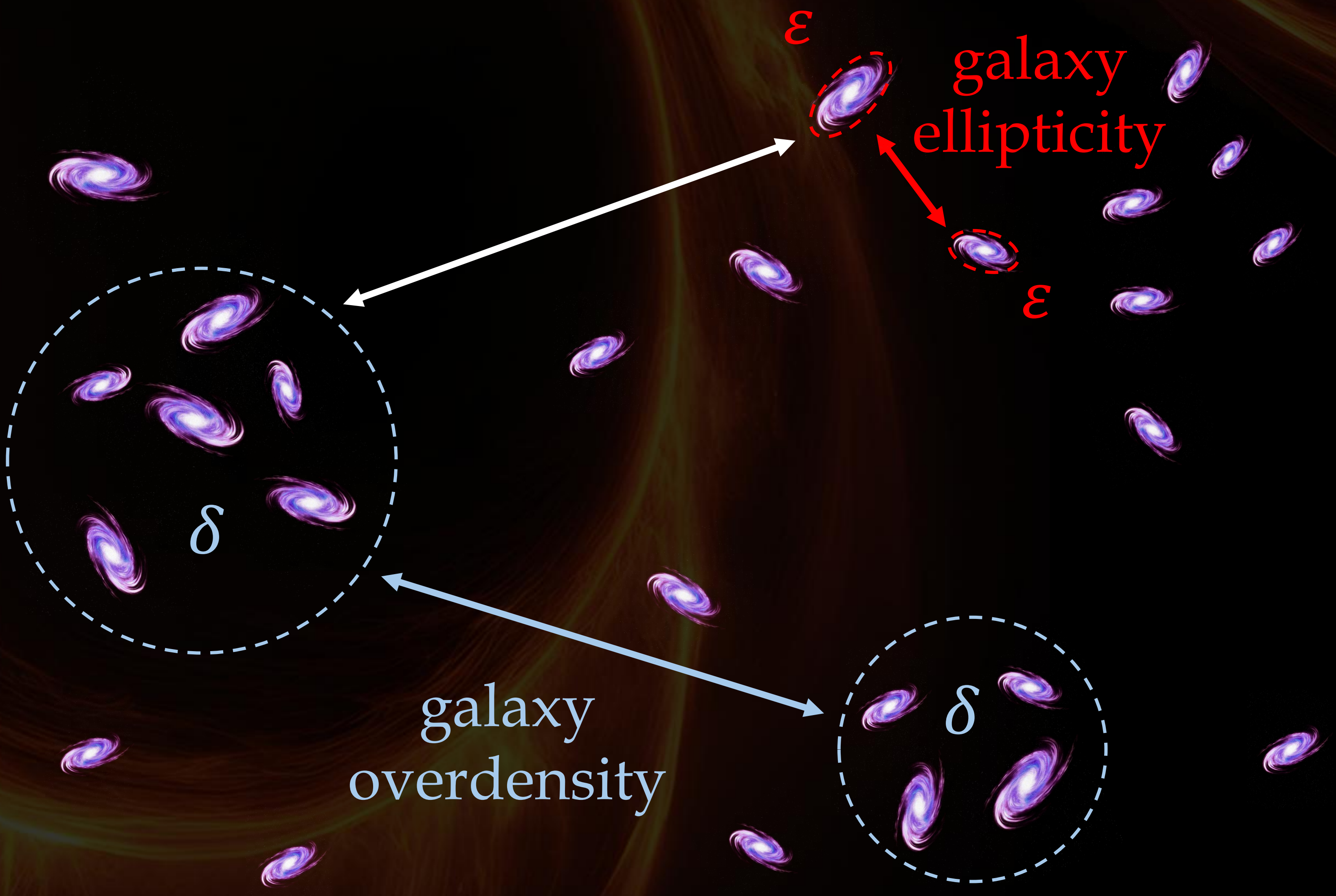


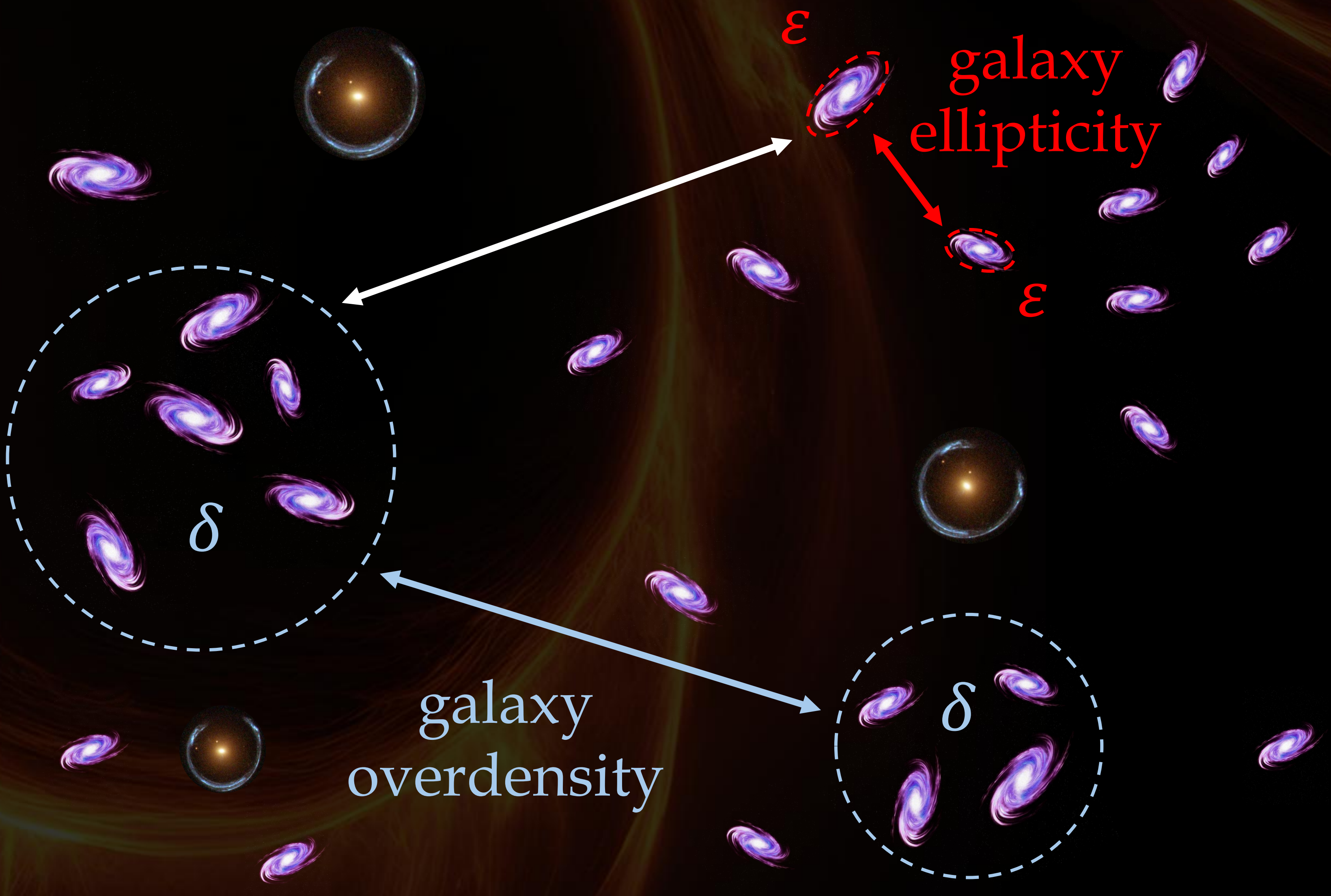
# The 3x2pt correlation scheme

$\langle \delta \times \delta \rangle$

$\langle \varepsilon \times \varepsilon \rangle$

$\langle \delta \times \varepsilon \rangle$

~$10^5$ lenses with *Euclid*



# The 6x2pt correlation scheme

$\langle \delta \times \delta \rangle$

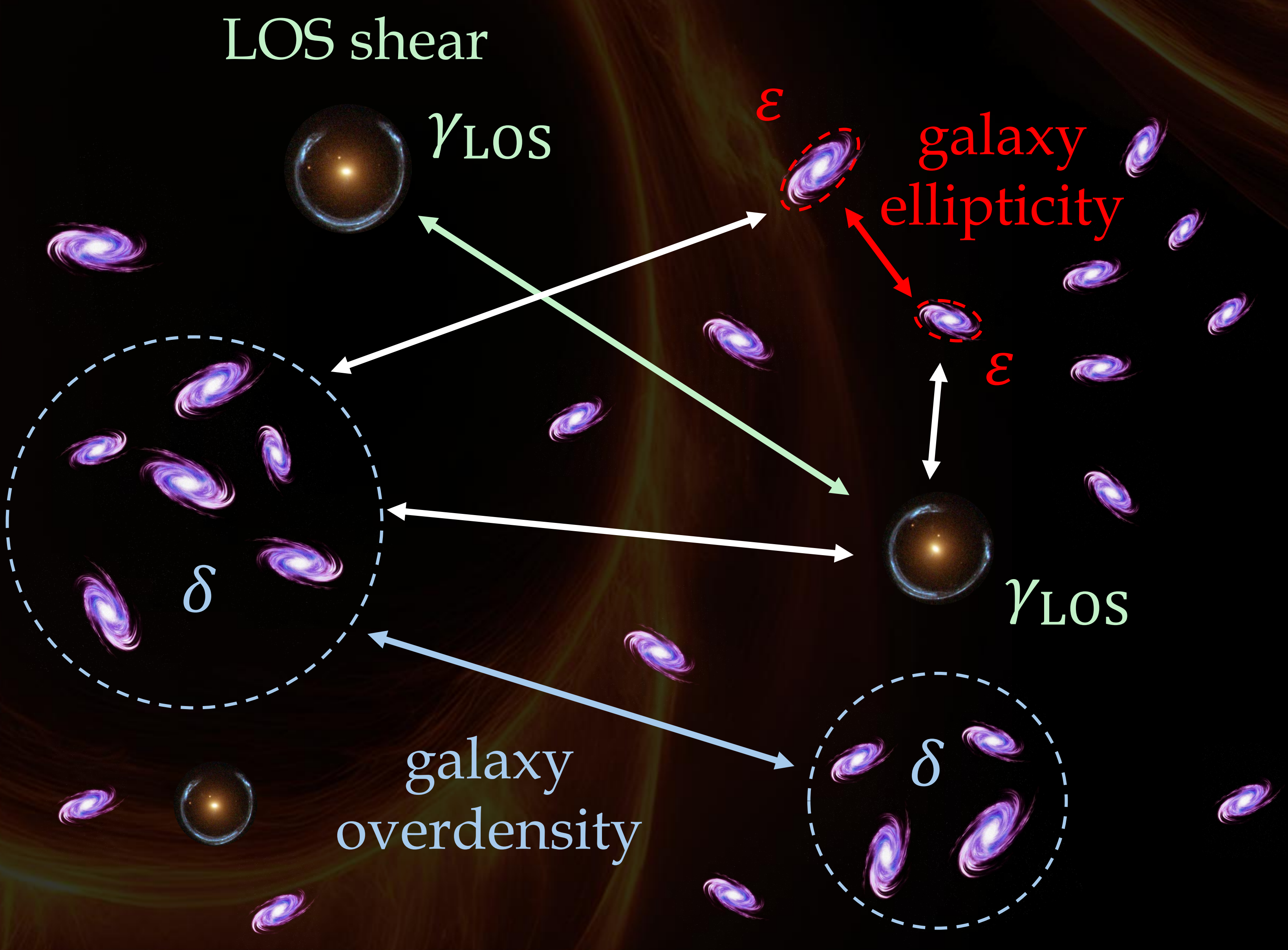
$\langle \varepsilon \times \varepsilon \rangle$

$\langle \delta \times \varepsilon \rangle$

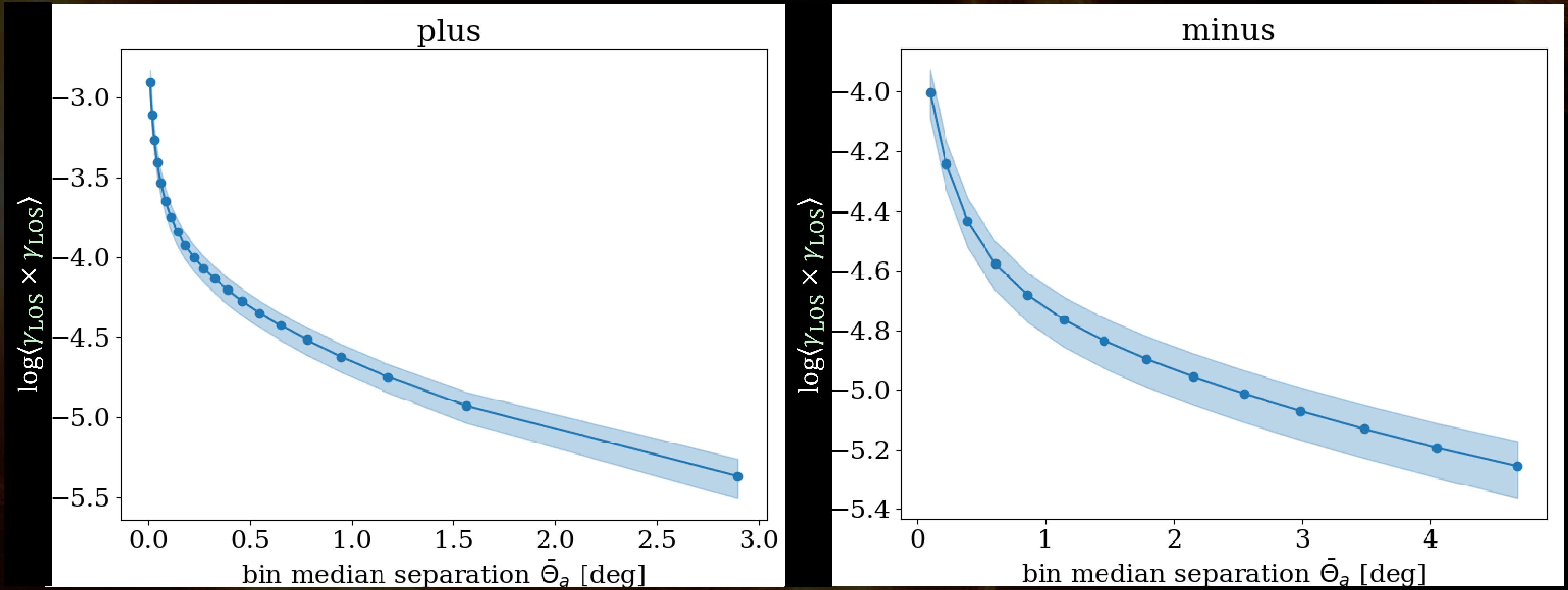
$\langle \gamma_{\rm LOS} \times \gamma_{\rm LOS} \rangle$

$\langle \gamma_{\rm LOS} \times \delta \rangle$

$\langle \gamma_{\rm LOS} \times \varepsilon \rangle$

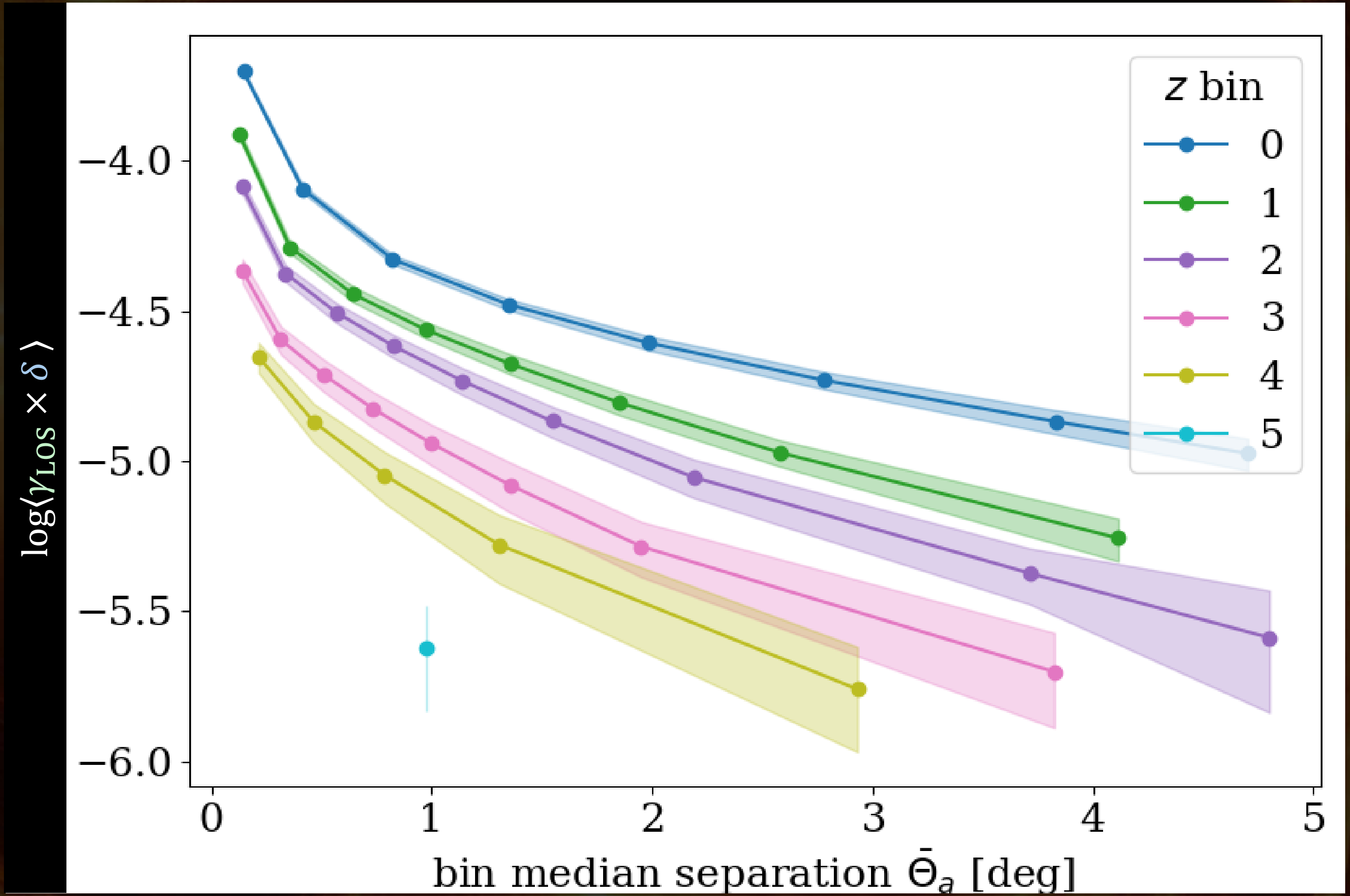


# LOS shear × LOS shear correlation function




$\sigma_{\rm LOS} = 1\%$ $\quad$ $N_{\rm lens} = 10^5$

# 3/3 – the line of sight as a cosmological probe

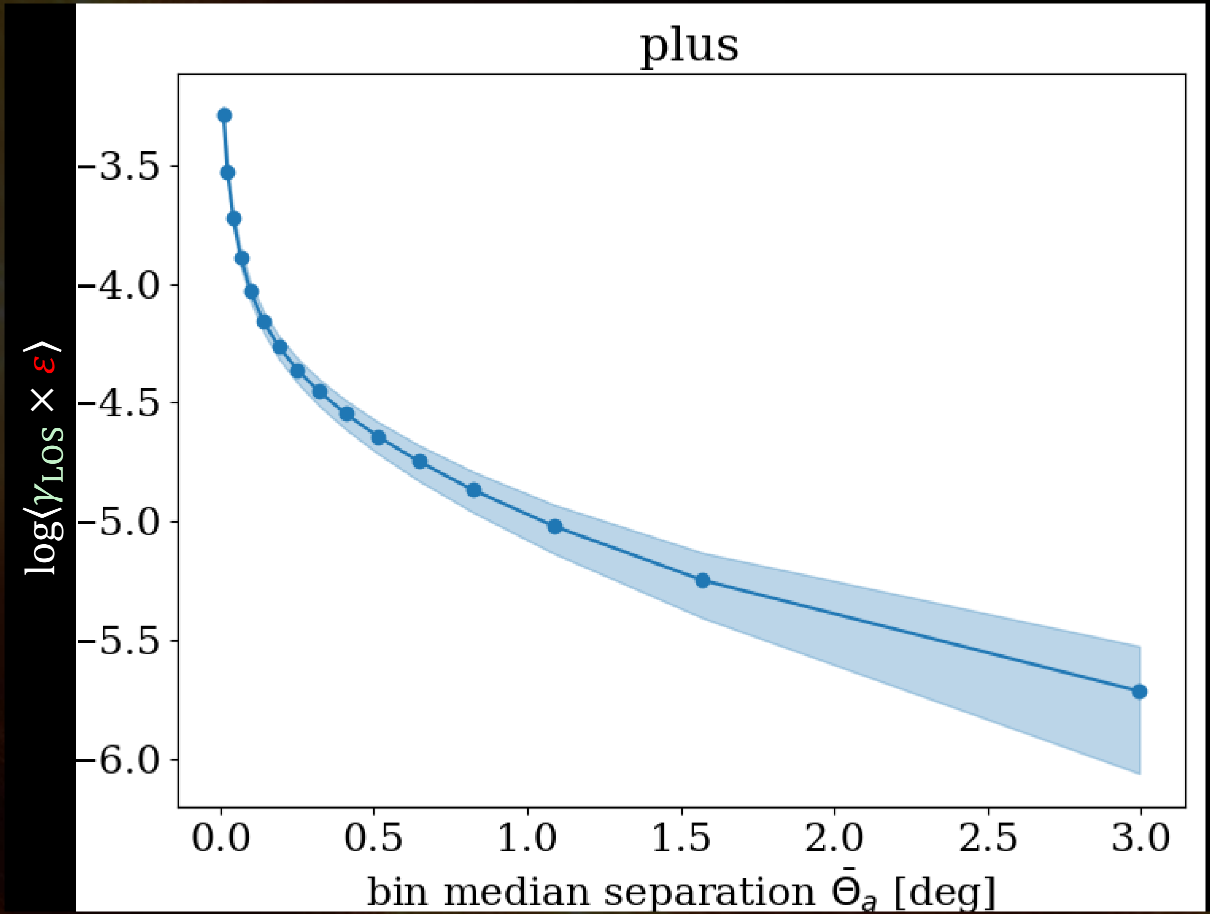


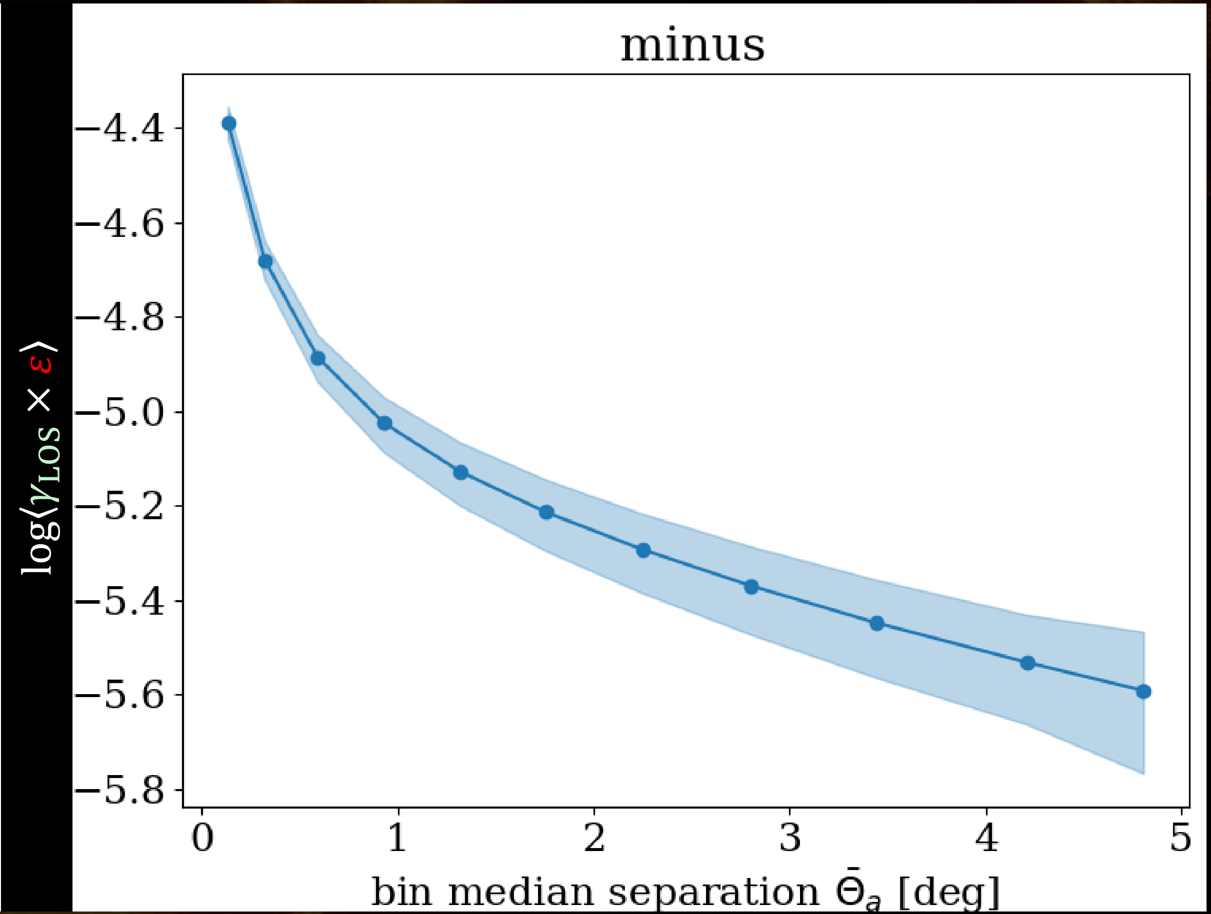

LOS shear ×
galaxy density
correlation
function

$\sigma_{\rm LOS} = 5\%$

$N_{\rm lens} = 10^5$

# LOS shear × galaxy ellipticity correlation function

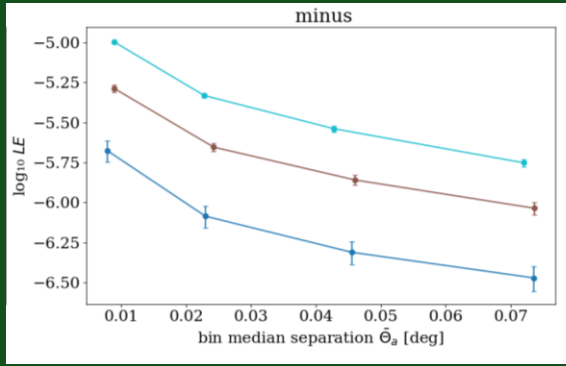



$\sigma_{\rm LOS} = 10\%$ $\quad N_{\rm lens} = 10^4$

## Observed Correlation Functions

How the relationship between observables changes as their separation increases



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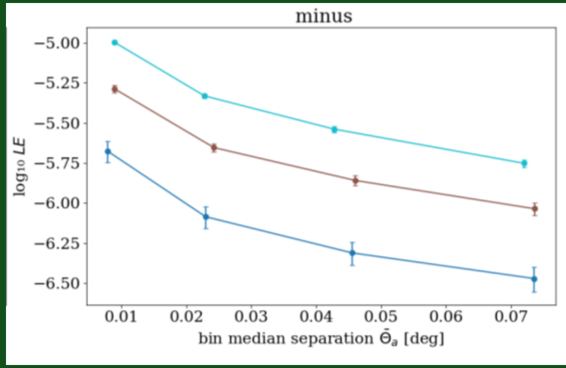
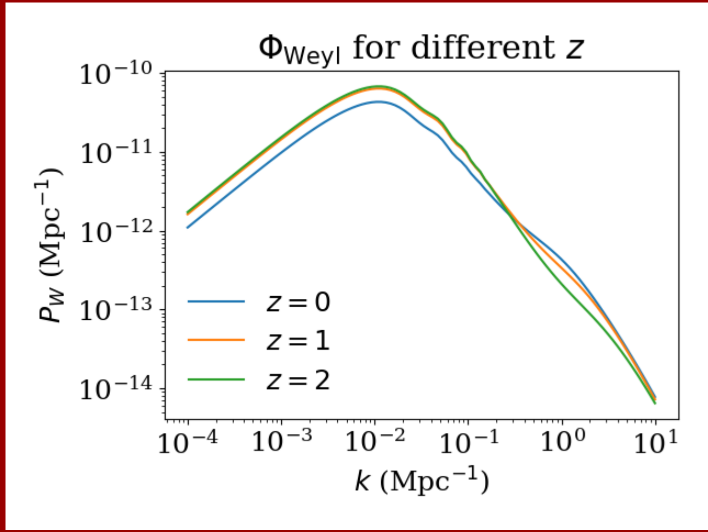
How the relationship between observables changes as their separation increases




## Matter power spectrum

How matter is distributed through the universe



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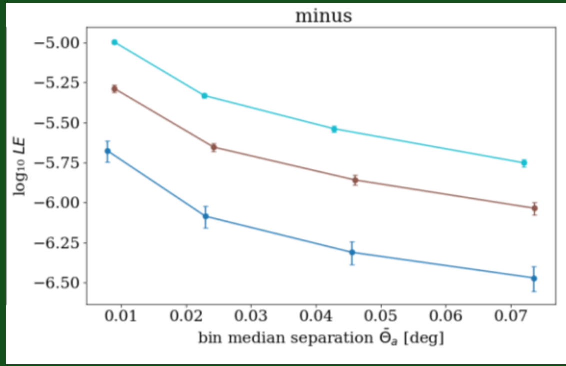


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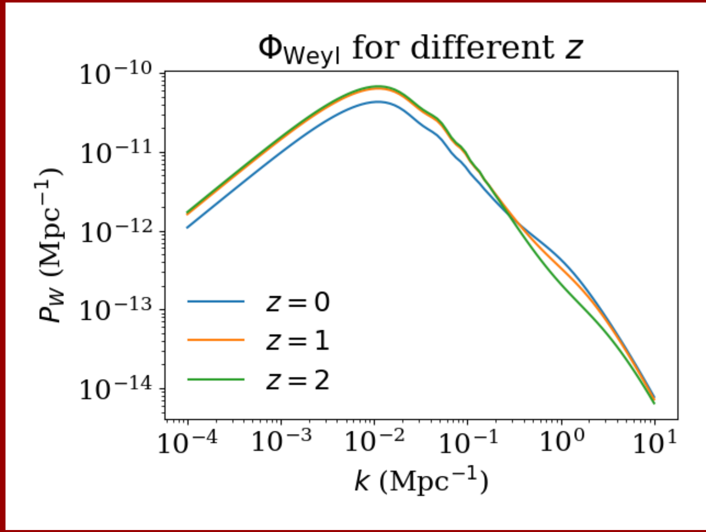


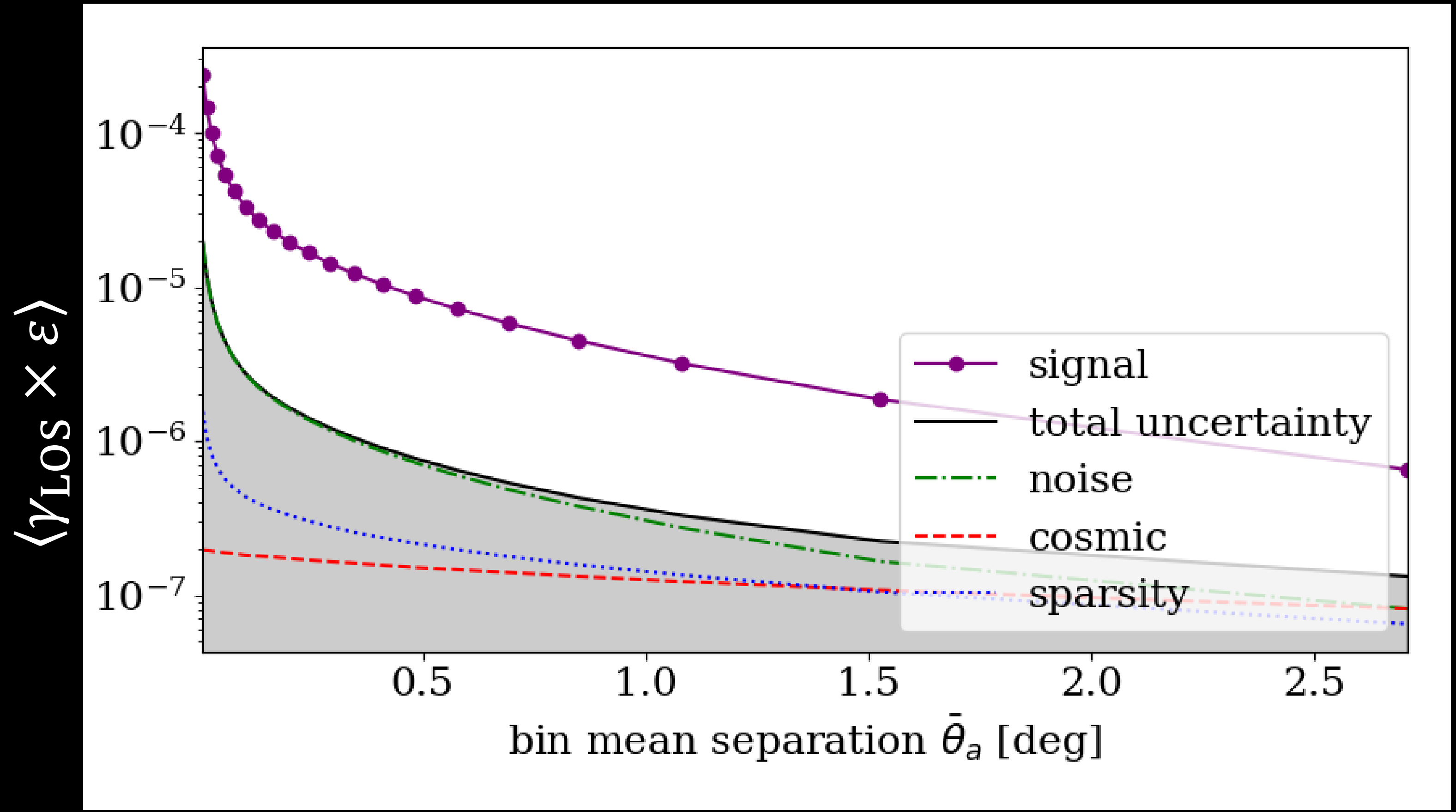

# Dark matter distribution

# Sources of uncertainty

noise

sparsity

cosmic



# Covariance matrices

**Cosmic covariance**

$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2}\Big[\int_{\Omega}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega}\,\xi^{+}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}|)\\&+\int_{\Omega}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega}\,\xi^{-}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}|)\cos 4(\psi''-\psi_{\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}})\\&+\int_{\Omega}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}''-\boldsymbol{l}|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'|)\\&+\int_{\Omega}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}''-\boldsymbol{l}|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}''+\boldsymbol{l}'}-\psi_{\boldsymbol{l}''-\boldsymbol{l}})\Big]\,,\end{aligned}\tag{1.1}$$

$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\xi^{+}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}-\boldsymbol{l}|)\cos 4(\psi'-\psi_{\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}})\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\xi^{-}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}|)\cos 4(\psi'-\psi'')\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}''|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\cos 4(\psi'-\psi_{\boldsymbol{l}-\boldsymbol{l}''})\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}''|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\cos 4(\psi'-\psi_{\boldsymbol{l}'+\boldsymbol{l}''})\Big]\,,\end{aligned}\tag{1.2}$$

$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\xi^{+}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l})\cos 4(\psi_{\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}}-\psi)\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\xi^{-}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\cos 4(\psi''-\psi)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l})\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}''|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}''}-\psi)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\\&+\int_{\Omega_{a''}}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega_{a''}}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'')\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\cos 4(\psi_{\boldsymbol{l}'+\boldsymbol{l}''}-\psi)\Big]\,,\end{aligned}\tag{1.3}$$

$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2}\Big[\int_{\Omega''_a}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega''_a}\xi^{+}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}|)\cos 4(\psi-\psi')\\&+\int_{\Omega''_a}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega''_a}\xi^{-}_{\gamma_{\mathrm{LOS}}}(l'')\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}|)\cos 4(\psi''+\psi_{\boldsymbol{l}''+\boldsymbol{l}'-\boldsymbol{l}}-\psi'-\psi)\\&+\int_{\Omega''_a}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega''_a}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}''|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\cos 4(\psi-\psi')\\&+\int_{\Omega''_a}\frac{\mathrm{d}^2\boldsymbol{l}''}{\Omega''_a}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}-\boldsymbol{l}''|)\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(|\boldsymbol{l}'+\boldsymbol{l}''|)\cos 4(\psi_{\boldsymbol{l}'+\boldsymbol{l}''}+\psi_{\boldsymbol{l}-\boldsymbol{l}''}-\psi-\psi')\Big].\end{aligned}\tag{1.4}$$

2

**Sparsity covariance**

$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2L}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{+}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\right]\\&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\frac{1}{2}\xi^{+}_{\gamma_{\mathrm{LOS}}}(|\boldsymbol{l}-\boldsymbol{l}'|)\xi^{+}_{\epsilon}(0)\right]\\&\quad+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{+}_{\epsilon}(0)\\&\quad+\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\left[3\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)+\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\right]\Big\}\,,\end{aligned}\tag{1.5}$$

$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2L}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{-}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi'-\psi_{\boldsymbol{l}-\boldsymbol{l}'})\right]\\&+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{-}_{\gamma_{\mathrm{LOS}}}(|\boldsymbol{l}-\boldsymbol{l}'|)\xi^{+}_{\epsilon}(0)\cos 4(\psi'-\psi_{\boldsymbol{l}-\boldsymbol{l}'})\right]\\&+\frac{2\delta_{aa'}\delta_{bb'}\Omega}{LG_b\Omega_a}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l).\,,\end{aligned}\tag{1.6}$$

$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2L}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{-}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi)\right]\\&+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{-}_{\gamma_{\mathrm{LOS}}}(|\boldsymbol{l}-\boldsymbol{l}'|)\xi^{+}_{\epsilon}(0)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi)\right]\\&+\frac{2\delta_{aa'}\delta_{bb'}\Omega}{LG_b\Omega_a}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\,,\end{aligned}\tag{1.7}$$

$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]\\&=\frac{1}{2L}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{+}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi-\psi')\right]\\&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l')+\xi^{+}_{\epsilon}(0)\xi^{+}_{\gamma_{\mathrm{LOS}}}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi-\psi')\right]\\&\quad+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\mathrm{LOS}}}(0)\xi^{+}_{\epsilon}(0)\\&\quad+\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\frac{\mathrm{d}^2\boldsymbol{l}'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)+\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\xi^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(l)\right]\Big\}.\end{aligned}\tag{1.8}$$

**Noise covariance**

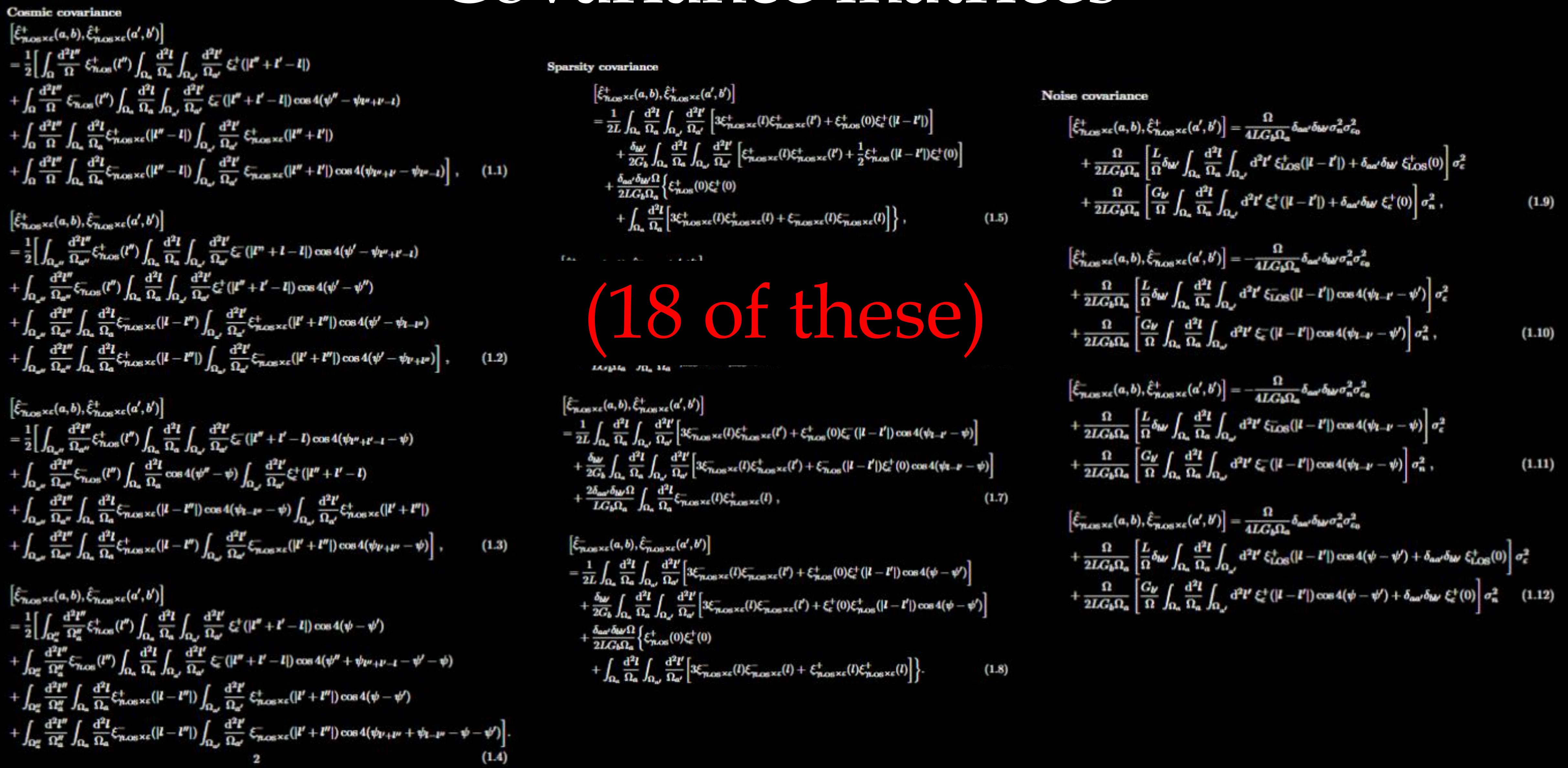
$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{+}_{\mathrm{LOS}}(|\boldsymbol{l}-\boldsymbol{l}'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\mathrm{LOS}}(0)\right]\sigma_\epsilon^2\\&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{+}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2\,,\end{aligned}\tag{1.9}$$

$$\begin{aligned}&\left[\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{-}_{\mathrm{LOS}}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi')\right]\sigma_\epsilon^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{-}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi')\right]\sigma_n^2\,,\end{aligned}\tag{1.10}$$

$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{-}_{\mathrm{LOS}}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi)\right]\sigma_\epsilon^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{-}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi_{\boldsymbol{l}-\boldsymbol{l}'}-\psi)\right]\sigma_n^2\,,\end{aligned}\tag{1.11}$$

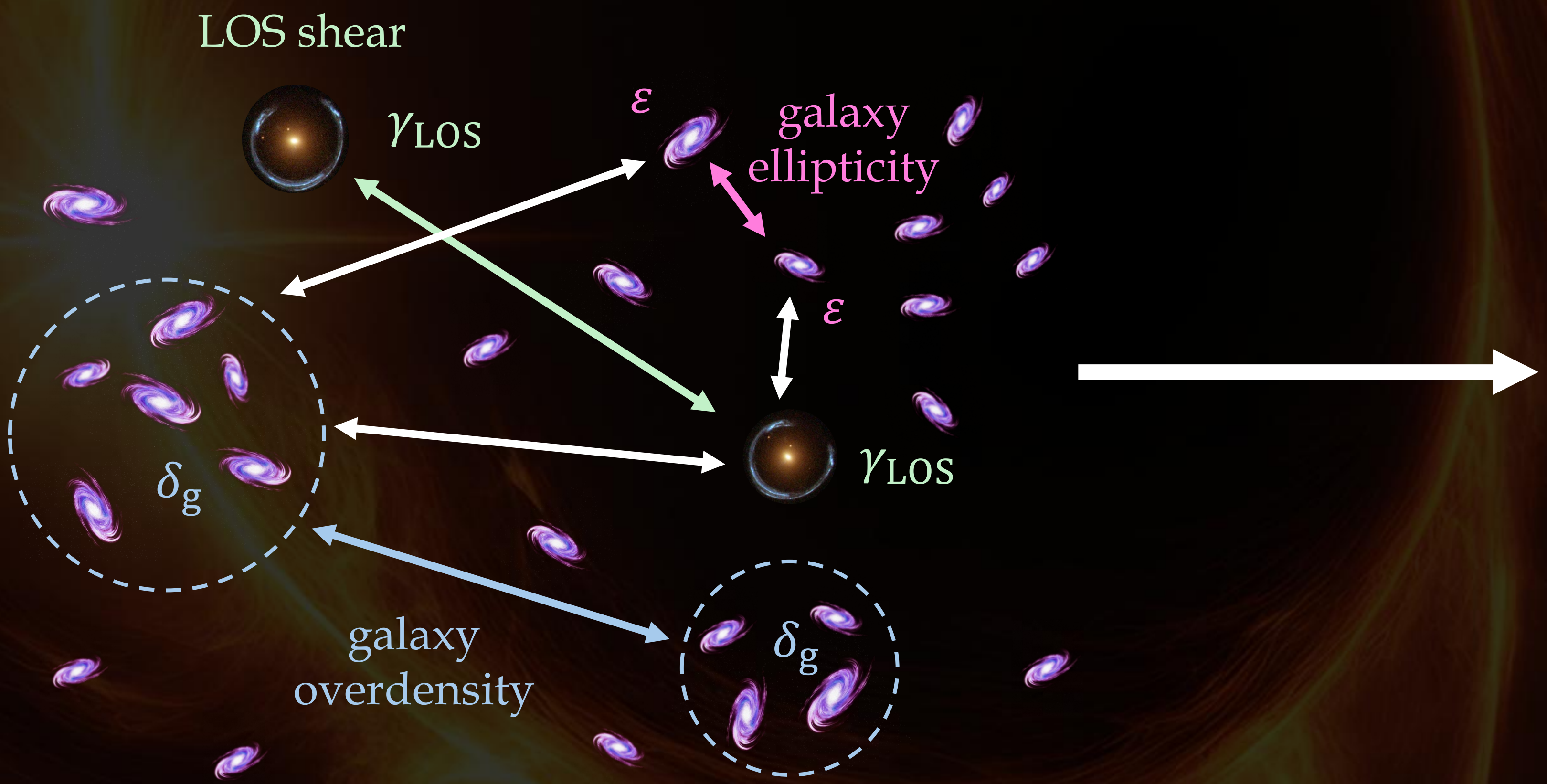
$$\begin{aligned}&\left[\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\mathrm{LOS}}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{+}_{\mathrm{LOS}}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\mathrm{LOS}}(0)\right]\sigma_\epsilon^2\\&+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{\mathrm{d}^2\boldsymbol{l}}{\Omega_a}\int_{\Omega_{a'}}\mathrm{d}^2\boldsymbol{l}'\,\xi^{+}_{\epsilon}(|\boldsymbol{l}-\boldsymbol{l}'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2\end{aligned}\tag{1.12}$$

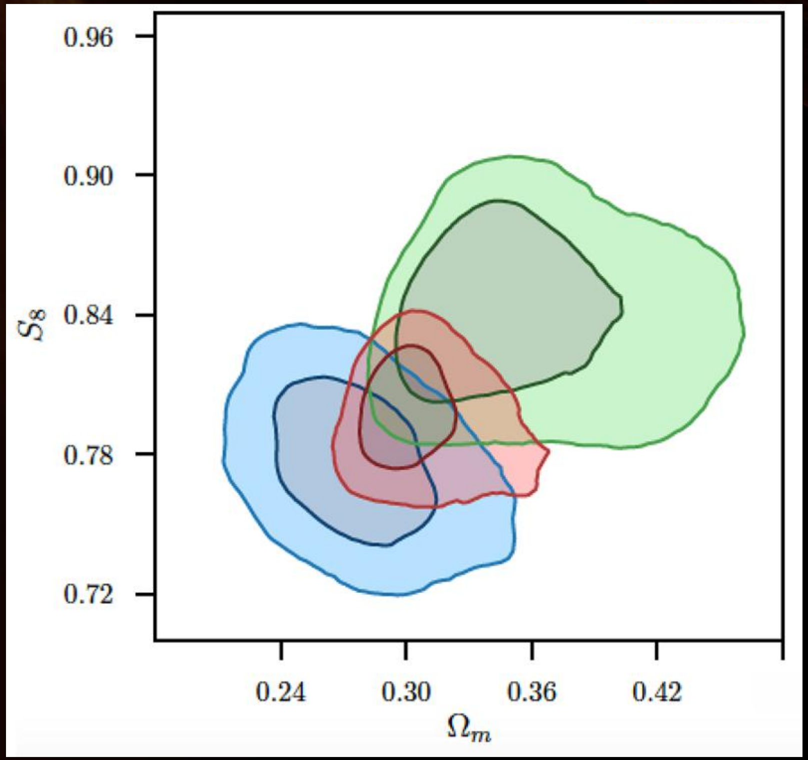
# Covariance matrices




(18 of these)

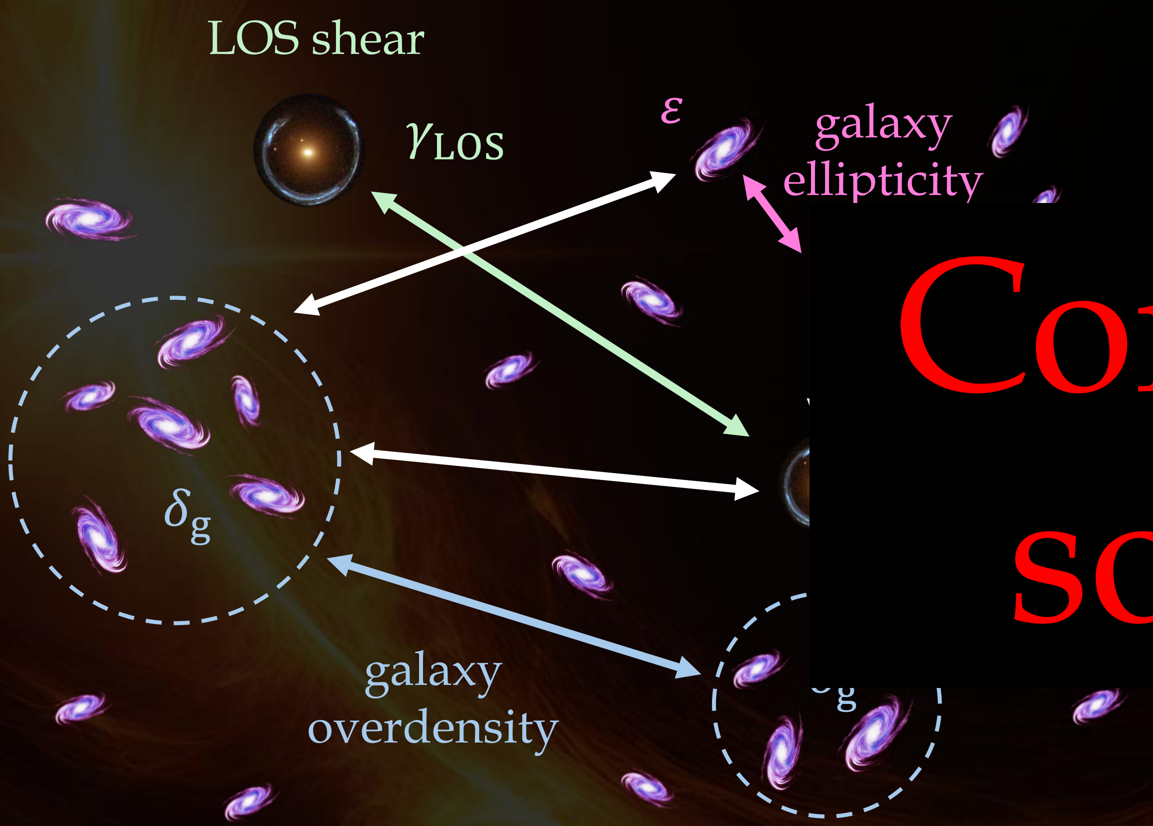
# The goal



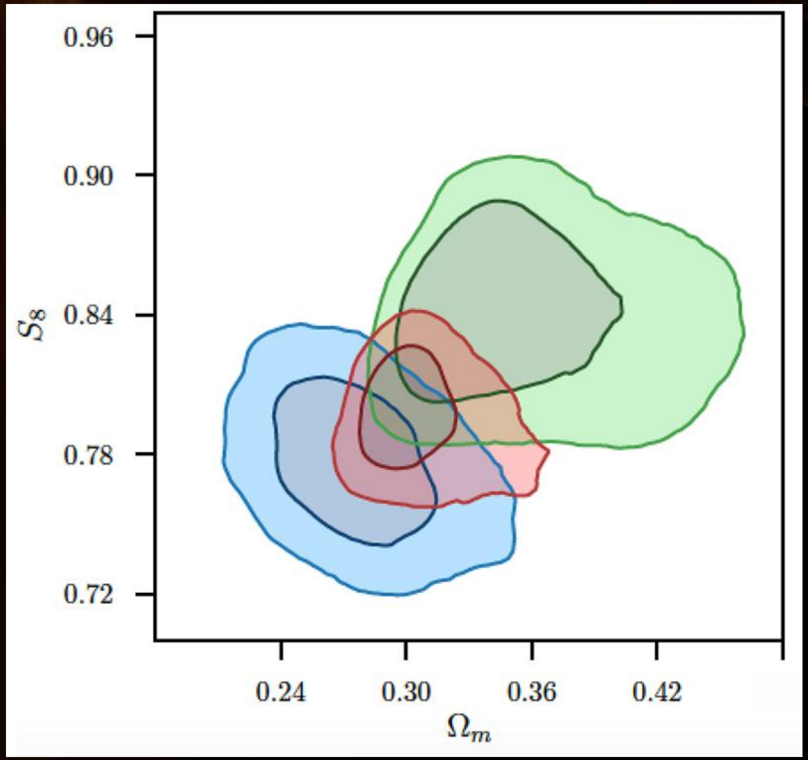




Credit: T. M. C. Abbott *et al.*, Phys. Rev. D (2018)

# The goal




**Coming soon!**




Credit: T. M. C. Abbott *et al.*, Phys. Rev. D (2018)

# Conclusions

- Gravitational lensing is a powerful observational tool for studying dark matter

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- Gravitational lensing is a powerful observational tool for studying dark matter
- Matter along the line of sight has a non-negligible effect on strong lensing observables
- Measurements of these effects offers a new way to probe the matter power spectrum

# Thanks for listening!

daniel.johnson@umontpellier.fr

**Further reading:**

Pierre Fleury, Julien Larena, Jean-Philippe Uzan,
*Line-of-sight effects in strong gravitational lensing*, JCAP 08 (2021) 024, arXiv:2104.08883

Natalie B. Hogg, Pierre Fleury, Julien Larena, Matteo Martinelli,
*Measuring line-of-sight shear with Einstein rings: a proof of concept*, MNRAS 520 (2023) 04, arXiv:2210.07210

Natalie B. Hogg, Anowar J. Shajib, Daniel Johnson, Julien Larena,
*Measuring line-of-sight shear with Einstein rings: a proof of concept*, MNRAS 520 (2023) 04, arXiv:2210.07210

Daniel Johnson, Pierre Fleury, Julien Larena, Lucia Marchetti,
*Foreground biases in strong gravitational lensing*, JCAP 10 (2024) 055, arXiv:2405.04194

Daniel Johnson, Thomas Collett, Tian Li, Pierre Fleury,
*Line-of-sight effects on double source plane lenses*, arXiv:2501.17153

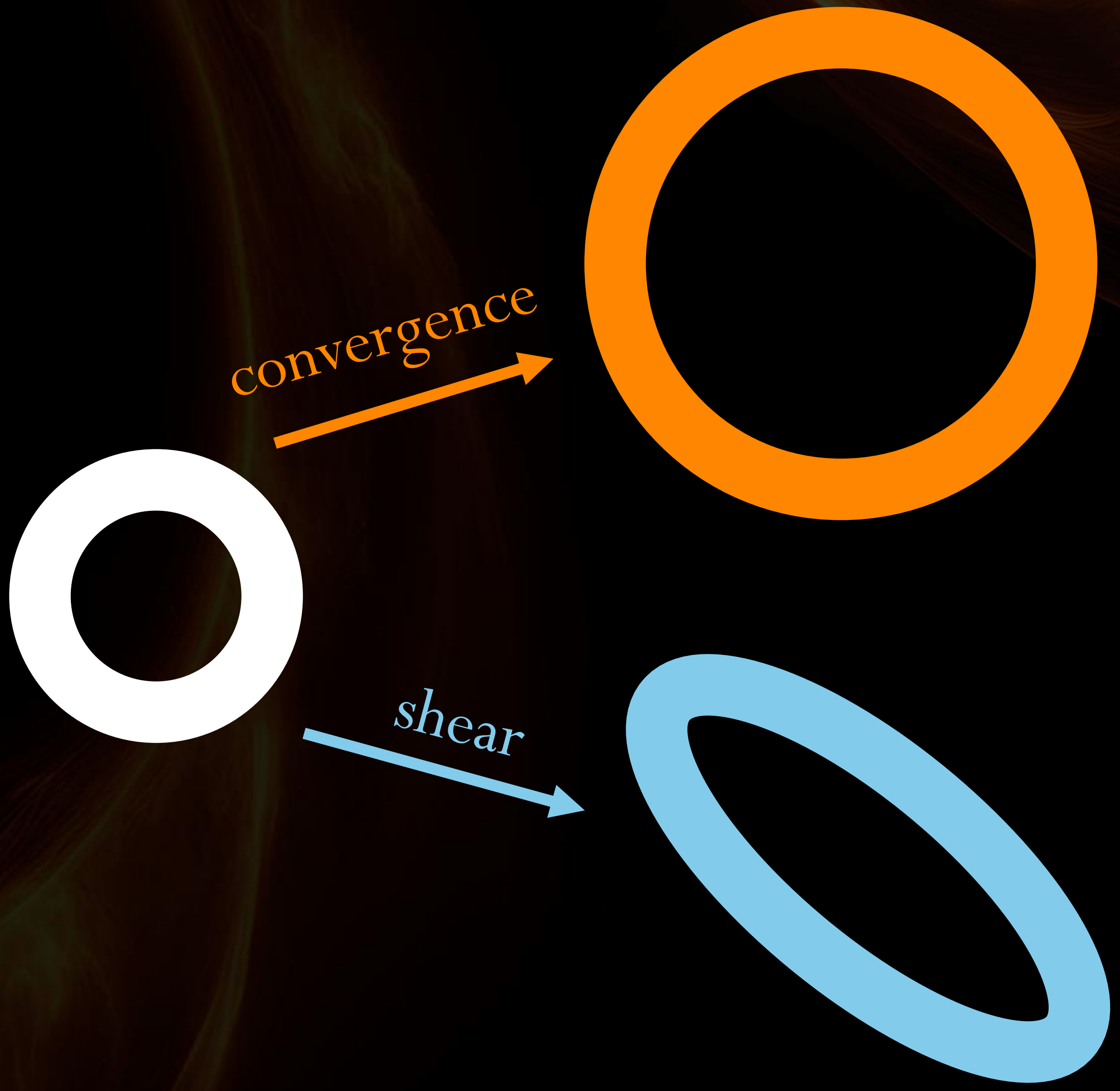
# Tidal weak lensing

**Convergence and shear**

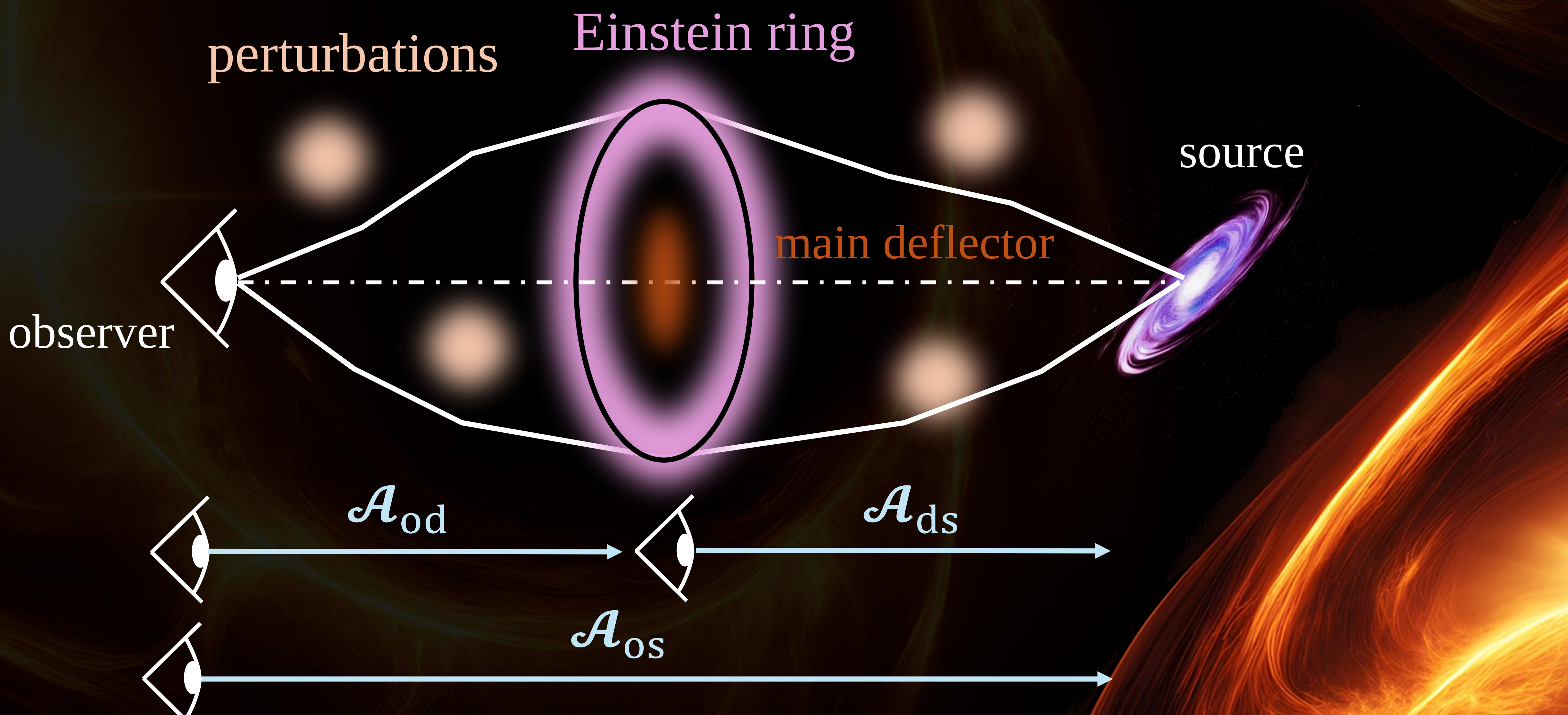
$$\beta = A\theta$$

source position

image position

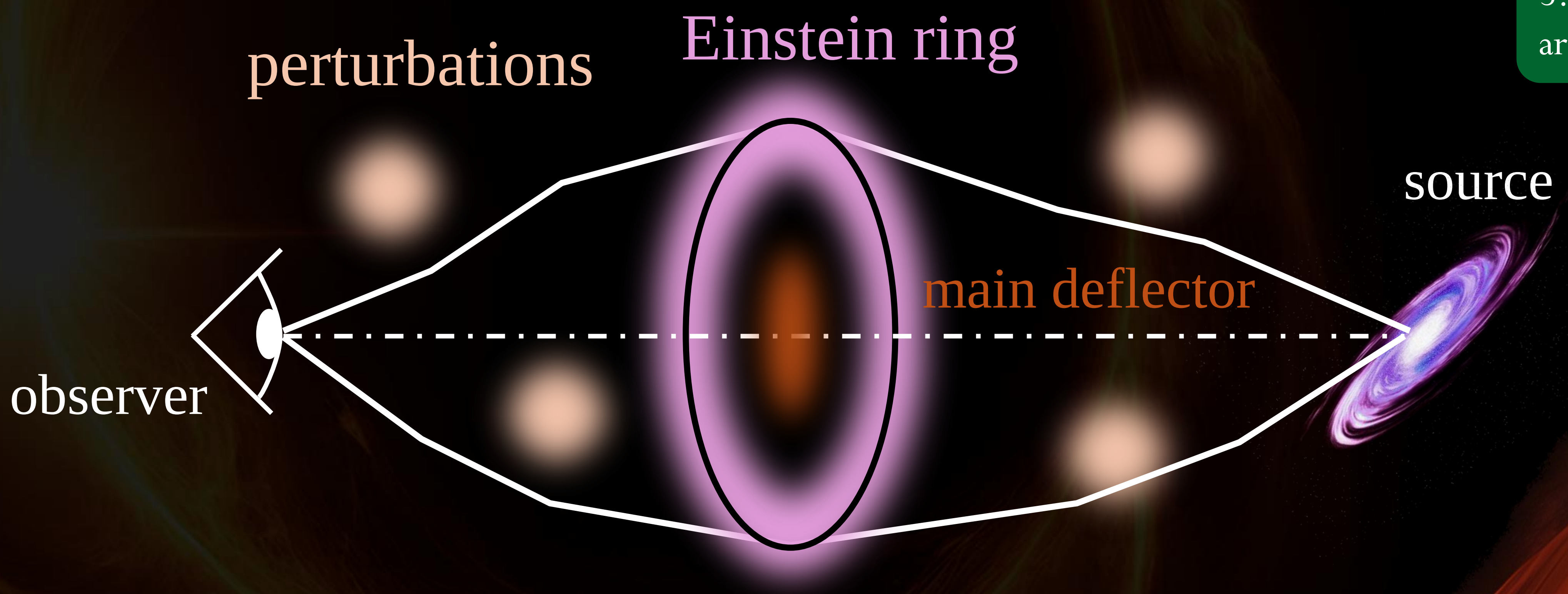


# Line-of-sight effects



# Line-of-sight effects

3. LOS effects
arXiv:2104.08883




$$\mathcal{A}_{\mathrm{LOS}} \approx \mathcal{A}_{\mathrm{od}} + \mathcal{A}_{\mathrm{os}} - \mathcal{A}_{\mathrm{ds}}$$

# The minimal lens model

Fleury et al. 2021, 2104.08883

$$\tilde{\beta} = \boldsymbol{\mathcal{A}}_{\mathrm{LOS}} \boldsymbol{\theta} - \frac{\mathrm{d}\psi_{\mathrm{eff}}}{\mathrm{d}\boldsymbol{\theta}}$$

(A single main lens + tidal line-of-sight effects)

# The minimal lens model

Fleury et al. 2021, 2104.08883

$$\tilde{\boldsymbol{\beta}} = \boldsymbol{\mathcal{A}}_{\rm LOS}\boldsymbol{\theta} - \frac{\mathrm{d}\psi_{\rm eff}}{\mathrm{d}\boldsymbol{\theta}}$$

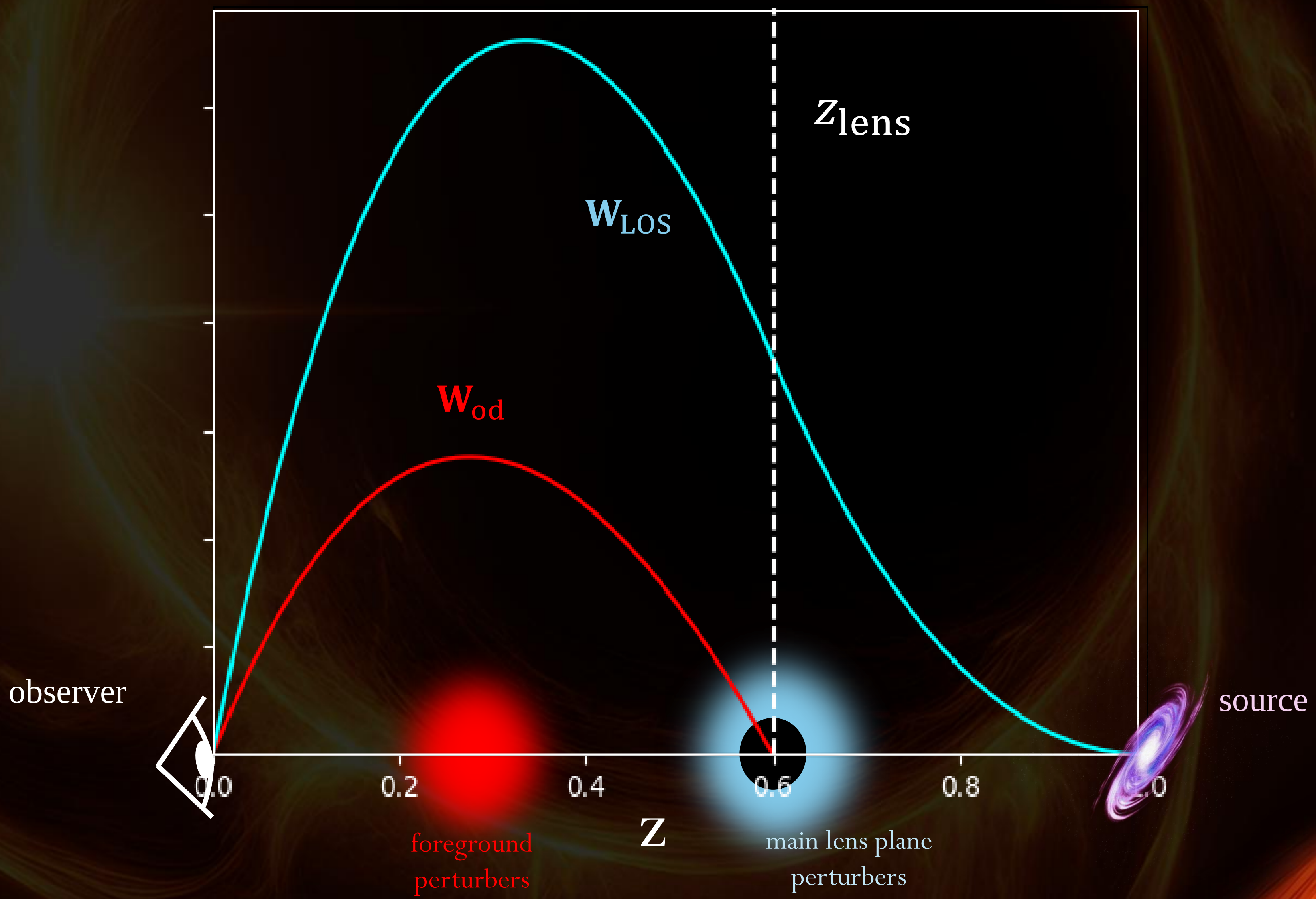
"External" convergence
and shear

$$\psi_{\rm eff}(\boldsymbol{\theta}) \equiv \psi[\boldsymbol{\mathcal{A}}_{\rm od}\boldsymbol{\theta}]$$

Foreground convergence
and shear

# Backup slides

# Weight functions

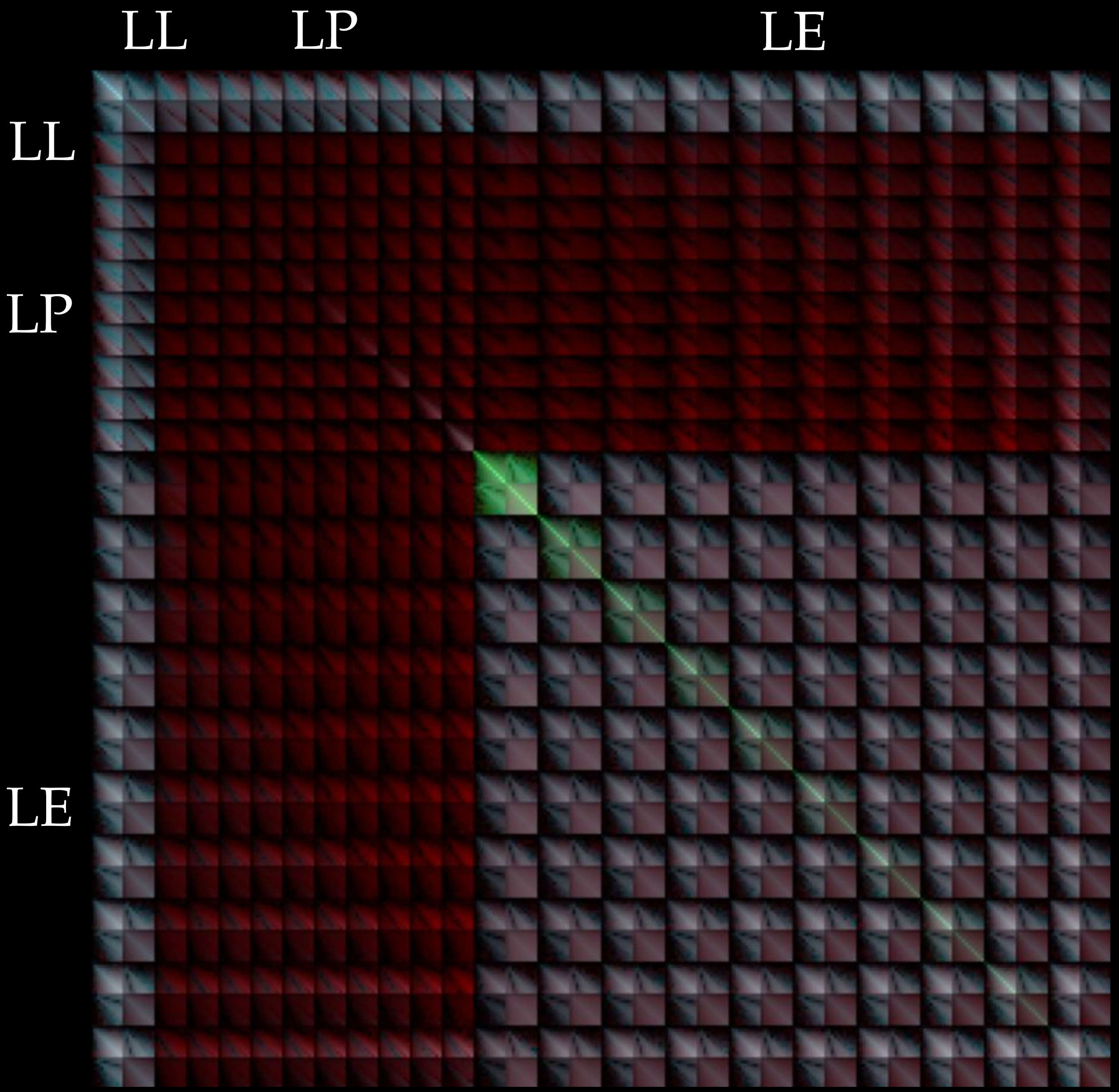


# Covariance matrices

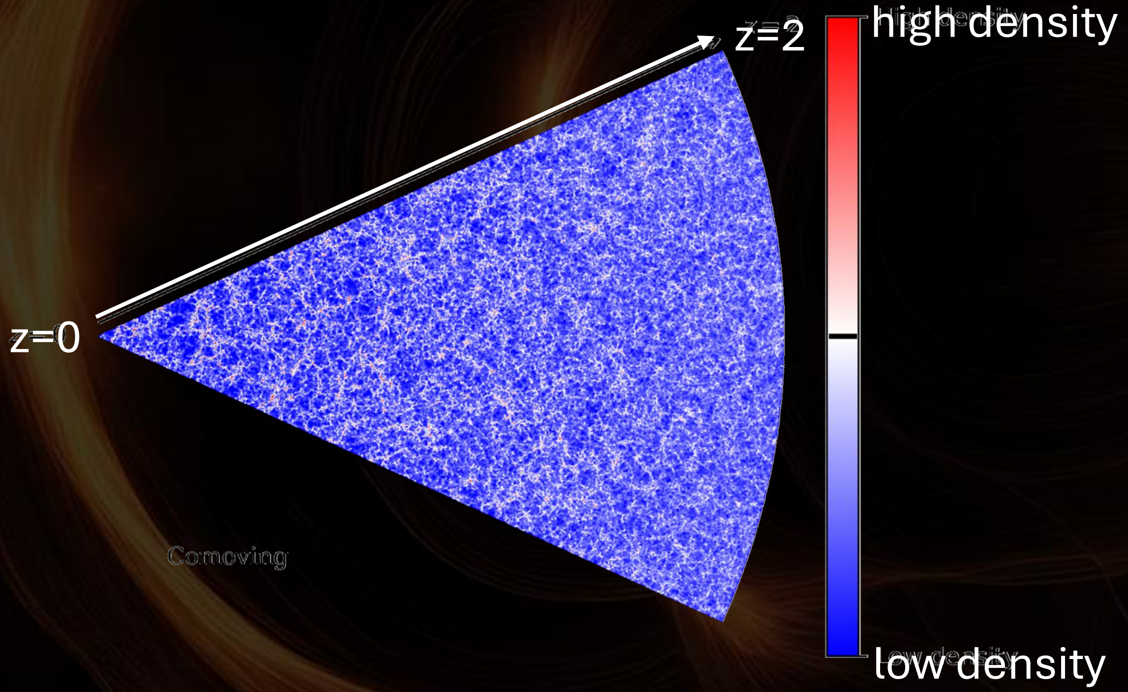
noise

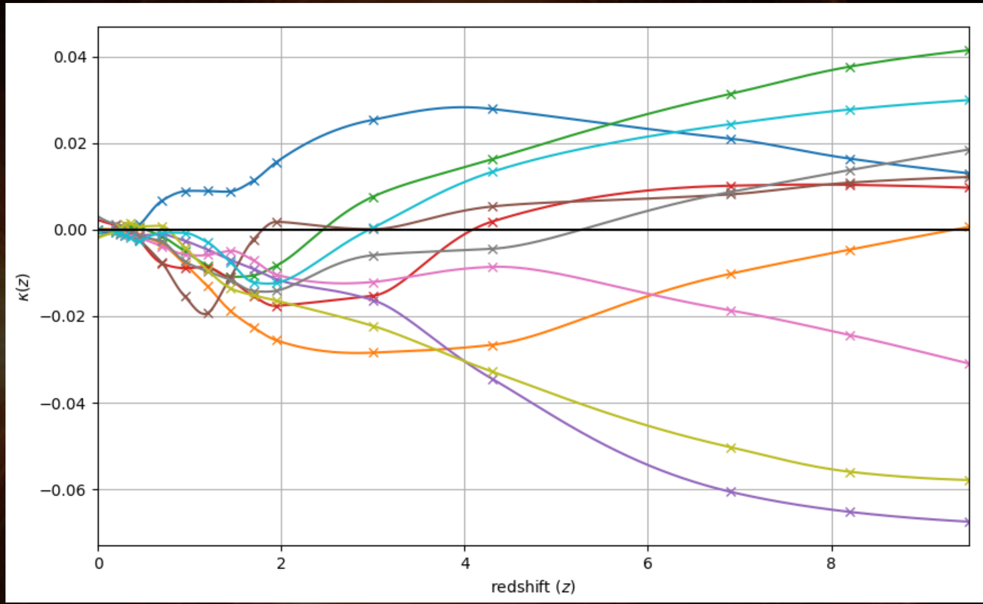
sparsity

cosmic



# Generating lines of sight




- Interpolate between datapoints
- Integrate to determine background terms

# Foreground shear effects (ellipticity is biased)

$$\varepsilon_{\mathrm{eff}} \equiv \varepsilon + 2(5-\gamma) g_{\mathrm{od}}$$

If the foreground shear is left out of a model, you will measure an effective ellipticity. However, other parameters should be unaffected